UNIT 3

DESIGN OF SPRINGS

3.1 INTRODUCTION

A spring is defined as an elastic body whose function is to distort when loaded and to recover its original shape when the load is removed.

The important applications of the springs are as follows :

(i) To apply forces as in brakes, clutches and spring loaded valves.

(ii) To measure forces as in spring balances.

(iii) To store energy as in watch springs.

(iv) To absorb shock and vibrations as in car springs and railway buffers.

(v) To control the motion by maintaining contact between two elements as in cams and followers.

3.2 TYPES OF SPRINGS

(1) **Helical Springs :** Helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile load. The helical springs have the following advantages :

  (i) These are easy to manufacture.

  (ii) These are available in wide range

  (iii) These are reliable.

  (iv) These have constant spring rate.

  (v) Their performance can be predicted more accurately.

  (vi) Their characteristics can be varied by changing dimensions.

(2) Conical and involute springs.

(3) Torsion springs.

(4) Laminated or leaf springs.

(5) Disc or bellevile springs.

(6) Special purpose springs.

3.3 TERMS USED IN COMPRESSION SPRINGS

(i) **Solid Length :** When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid.

Solid length = i'd Where i' = Total number of turns and

d = diameter of spring wire

(ii) **Free Length :** The free length of a compression spring is the length of the spring in the free or unloaded condition

free length $l_o \geq (i + n)\, d + y + a$

where a = clearance

i = number of active turns or coils

y = maximum deflection

(iii) **Spring Index :** The spring index is defined as the ratio of mean diameter of coil to the diameter of wire.

$$\text{Spring index } c = \frac{D}{d} \quad \text{where } D = \text{Mean coil diameter.}$$

(iv) **Spring Rate :** The spring rate is defined as the load required per unit deflection of the spring.

$$\text{Spring rate } F_o = \frac{F}{y} \quad \text{where } y = \text{axial deflection.}$$

(v) **Pitch :** The pitch of the coil is defined as the axial distance between adjacent coils in un-compressed state

$$\text{Pitch } p = \frac{l_o - 2d}{i} \text{ for squared and ground end.}$$

Where i = number of active turns and l_o = free length.

3.4 SPRING MATERIALS

Four basic varieties of steel wires are used in springs. They are,

(i) Patented and cold drawn steel wires (unalloyed)

(ii) Oil-hardened and tempered spring steel wires and valve spring wires (unalloyed)

(iii) Oil hardened and tempered steel wires (alloyed)

(iv) Stainless steal spring wires for normal corrosion resistance.

The patented and cold drawn steel wires are mainly used in springs subjected to static forces. Four grades of this wire are commonly used. Springs subjected to static or low load cycles, grade 1 is used. Springs subjected to moderate-load cycles, grade 2 is used. Springs subjected to moderate dynamic loads, grade 3 is used. Springs subjected to severe stresses, grade 4 is used.

For unalloyed, oil hardened and tempered spring steel wire and value spring wires two grades are used. They are SW and VW. Springs subjected to moderate fluctuating stresses grade SW is used and grade VW is recommended for springs subjected to high magnitude of fluctuating stresses.

For higher temperature applications alloyed varieties of oil hardened and tempered steel wires are used. Stainless steel springs are ideal to work in steam or some other corrosive medium because of its excellent corrosion resistance. Nonferrous materials such as brass, phosphor-bronze, silicon-bronze, monel and beryllium copper are also used as spring wires. Some of the cold-wound helical spring wires are music wire, oil-tempered wire, hard-drawn spring wire, chrome silicon, chrome vanadium, silicon manganese and stainless steel.

3.5 STRESS IN HELICAL SPRINGS OF CIRCULAR WIRE

Consider a helical compression spring made of circular wire and subjected to an axial load 'F' as shown in Fig. 3.1 a

Let, D = Mean diameter of coil

d = diameter of spring wire

i = number of active coils.

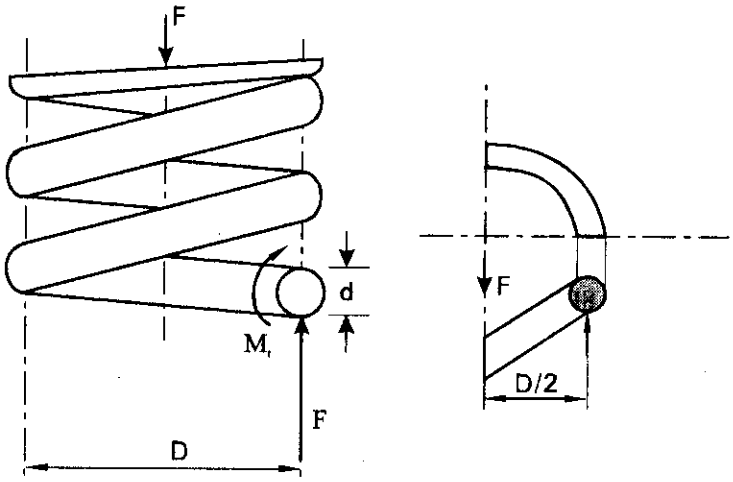
G = Modulus of rigidity.

F = Axial load on the spring

τ = Maximum shear stress on the spring

c = Spring Index

y = Deflection of the spring



(a) *(b)*

Fig. 3.1

Now consider a quadrant of a coil of a round wire spring as shown in Fig. 3.1b. The load 'F' acting along the axis of the spring which has a mean diameter 'D' produces a torsional moment 'T' or M_t.

$$\therefore\ M_t \text{ or } T = F.\frac{D}{2}$$

This torsional moment is also equal to $\frac{\pi}{16}\tau_1 d^3$

$$\therefore \ T = \frac{FD}{2} = \frac{\pi}{16}\tau_1 d^3$$

$$\therefore \ \tau_1 = \frac{8FD}{\pi d^3}$$ = stress produced in a member twisted by a couple in pure shear.

This torsional shear stress diagram is shown in Fig. 3.1 (c).

In addition to the torsional shear stress the following stresses are also act on the wire

(i) Direct stress due to load 'F'.

(ii) Stress due to curvature of wire.

Direct shear stress due to load 'F' $= \frac{\text{Load}}{\text{c / sArea of wire}}$

$$\text{i.e., } \tau_2 = \frac{F}{\frac{\pi}{4}d^2}$$

$$\therefore \ \tau_2 = \frac{4F}{\pi d^2}$$

The direct shear stress diagram is shown in Fig. 3.1 (d)

$\therefore$ The resultant shear stress induced in the wire $\tau = \tau_1 + \tau_2$

$$\text{i.e., } \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Positive sign is used for inner edge of wire and negative sign is used for outer edge of wire.

$\therefore$ Maximum shear stress induced in the wire $\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$

$$= \frac{8FD}{\pi d^3}\left[1 + \frac{d}{2D}\right] = \frac{8FD}{\pi d^3}\left[1 + \frac{1}{2c}\right] \qquad \left(\because c = \frac{D}{d}\right)$$

$$\therefore \ \tau = \frac{8FD}{\pi d^3}.k_s \qquad \text{---- (A)}$$

where $k_s = 1 + \frac{1}{2c}$ = Shear stress factor.

The resultant diagram of torsional shear stress and direct shear stress is shown in Fig. 3.1 (e).

Form the above equation it can be observed that the effect of direct shear stress i.e., $\frac{8FD}{\pi d^3}.\frac{1}{2c}$ is appreciable for springs of small spring Index 'c'. Also the effect of wire curvature is neglected in equation (A).

In order to consider the effect of both direct shear as well as curvature of the wire, Wahl's stress factor 'k' introduced by 'A.M. Wahl' may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress is shown in Fig. 3.1 (f)

$\therefore$ Maximum shear stress induced in the wire $\tau = \frac{8FD}{\pi d^3} .k$

$$\text{where } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \text{Wahl's stress factor.}$$

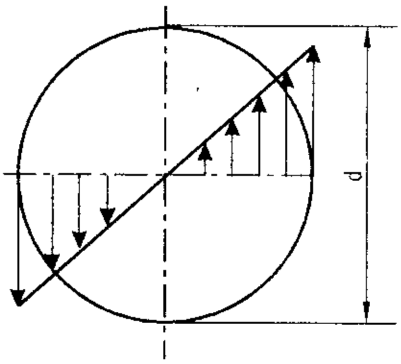


Fig. 3.1 c : Torsional shear Stress diagram

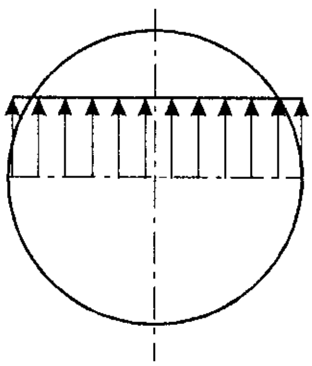


Fig. 3.1 d : Direct shear Stress diagram

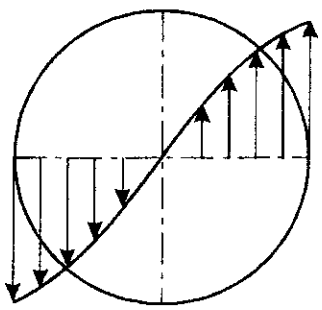


Fig. 3.1 e : Resultant torsional and direct shear shear stress diagram

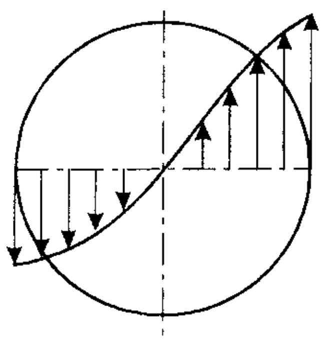


Fig. 3.1 f : Resultant torsional direct and curvature shear stress diagram

The values of k for a given spring index may be obtained from the graph shown in Fig. 20.5 (DDHB). From that we can see, the Wahl's factor increases very rapidly as the spring index decreases.

The Wahl's stress factor may be considered as composed of two sub-factors k_S and k_c

$$\therefore \; k = k_S . k_c$$

where k_S = Stress factor due to shear.

k_c = Stress concentration factor due to curvature.

3.6 DEFLECTION OF HELICAL SPRING OF CIRCULAR CROSS SECTION WIRE

Total length of wire l = length of one coil × Number of active coils

$$\therefore \; l = \pi D i$$

$\therefore$ Axial deflection of spring $y = \theta \frac{D}{2}$ where θ = angular deflection.

We know,

$$\frac{T}{J} = \frac{\tau}{\frac{d}{2}} = \frac{G\theta}{l}$$

$$\therefore \; \theta = \frac{Tl}{GJ} = \frac{\left(F.\frac{D}{2}\right)(\pi D i)}{G.\frac{\pi}{32}d^4}$$

$\therefore$ Angular deflection $\boldsymbol{\theta} = \mathbf{\frac{16FD^2.i}{G.d^4}}$

Hence axial deflection $y = \theta.\frac{D}{2} = \frac{16FD^2.i}{G.d^4}.\frac{D}{2}$

$$\therefore \; y = \mathbf{\frac{8FD^3.i}{d^4.G}}$$

$$\text{Stiffness } F_o = \frac{F}{y} = \frac{F}{\frac{8FD^3.i}{d^4.G}}$$

$$\therefore \; F_o = \mathbf{\frac{d^4.G}{8D^3.i}}$$

3.7 ECCENTRIC LOADING OF SPRINGS

When the load is offset by a distance 'e' from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor $\frac{D}{2e + D}$ where D is the mean diameter of coil.

3.8 SURGE IN SPRINGS

When one end of a helical spring is resting on a rigid support and other end is loaded suddenly, then all the coils of the spring will not deflect equally because some time is required for the propagation of stress along the spring wire. In the beginning, the end of the coil of the spring in contact with the applied load takes up whole of the deflection and then it transmits a large part of its deflection to the adjacent coils. In this way a wave of compression propagates through the coils to the supported end from where it is reflected back to the deflected end. If the applied load is of fluctuating type as in the case of valve spring in internal combustion engine and if the time internal between the load application is equal to the time required for the wave to travel from one end to other end, then resonance will occur. This results in very large deflection of the coils and correspondingly very high stresses. Under these conditions the spring may fail. This phenomenon is called 'Surge'.

Surging may occur in helical spring which have loads applied repetitively at a rate close to the natural frequency of the spring. To avoid this it is advisable that the natural frequency of the spring be atleast 20 times the frequency of the applied load.

The natural frequency for springs between two plates is given by,

$$f = \frac{d}{iD^2}\sqrt{\frac{G}{w}} \text{ Hz} \qquad \text{(20.78 old DDHB)}$$

Or Frequency of vibration of valve spring per minute $f_n = 84.627\sqrt{\frac{k_o}{W}}$ [20.77a New DDHB]

The surge in springs may be eliminated by using the following methods.

(i) By using friction dampers on the centre coils so that wave propagation dies out.

(ii) By using springs of high natural frequency.

(iii) By using springs having pitch of the coils near the ends different from than at the centre to have different natural frequencies.

3.9 EXPRESSION FOR STRAIN ENERGY STORED IN A BODY WHEN THE LOAD IS APPLIED GRADUALLY

The strain energy stored in a body is equal to the work done by the applied load in stretching the body. Fig. 2.2 shows load extension diagram of a body under tensile load up to elastic limit. The tensile load F increases gradually from zero to the value of F and the extension of the body increases from zero to the value of y. The load F performs work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load F is removed.

Let F = Gradually applied load

y = Extension of the body (spring)

A = Cross-sectional area

l = Length of body

V = Volume of the body

E = Young's modulus

U = Strain energy stored in the body

σ = Stress induced in the body

Now, work done by the load = Area of load extension curve

$$= \text{Area of } \Delta^{le}\ OAB = \frac{1}{2}Fy \qquad \text{---- (i)}$$

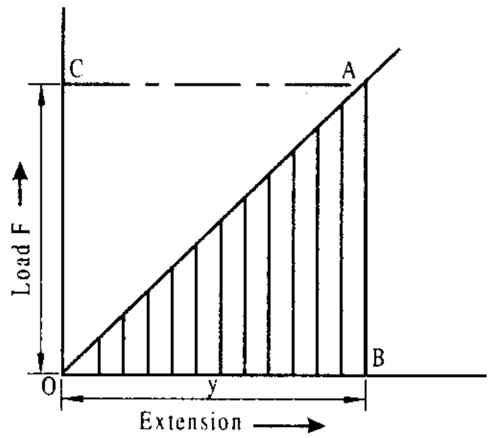


Fig. 3.2

$$\text{Load F} = \text{Stress} \times \text{Area} = \sigma A$$

$$\text{Extension y} = \text{strain} \times \text{length}$$

$$= \frac{\text{stress}}{E} \times l = \frac{\sigma}{E}.l \quad \left(\because \frac{\text{stress}}{\text{strain}} = E\right)$$

Substituting the values of F and y in equation (i)

$$\text{Work done by the load} = \frac{1}{2}\sigma A \times \frac{\sigma}{E} \times l = \frac{1}{2}\frac{\sigma^2}{E}.A.l$$

$$= \frac{\sigma^2}{2E}.V \quad [\because \text{volume } V = Al]$$

Since work done by the load in stretching the body is equal to the strain energy stored in the body,

$$\therefore \text{ Strain energy stored in the body } U = \frac{1}{2} Fy = \frac{\sigma^2}{2E}.V \qquad \text{---- (ii)}$$

3.9.1 Proof Resilience

The maximum energy stored in the body without permanent deformation [i.e., up to elastic limit] is known as proof resilience. Hence in equation (ii) if σ is taken at elastic limit, then we will get proof resilience.

$$\therefore \text{ Proof resilience} = \frac{\sigma^2}{2E} \times \text{volume where } \sigma = \text{stress at the elastic limit.}$$

$$\text{Modulus of resilience} = \text{strain energy per unit volume} = \frac{\text{Total strain energy}}{\text{Volume}}$$

$$= \frac{\frac{\sigma^2}{2E}.V}{V} = \frac{\sigma^2}{2E}$$

3.10 SYMBOLS USED IN HELICAL COMPRESSION SPRING

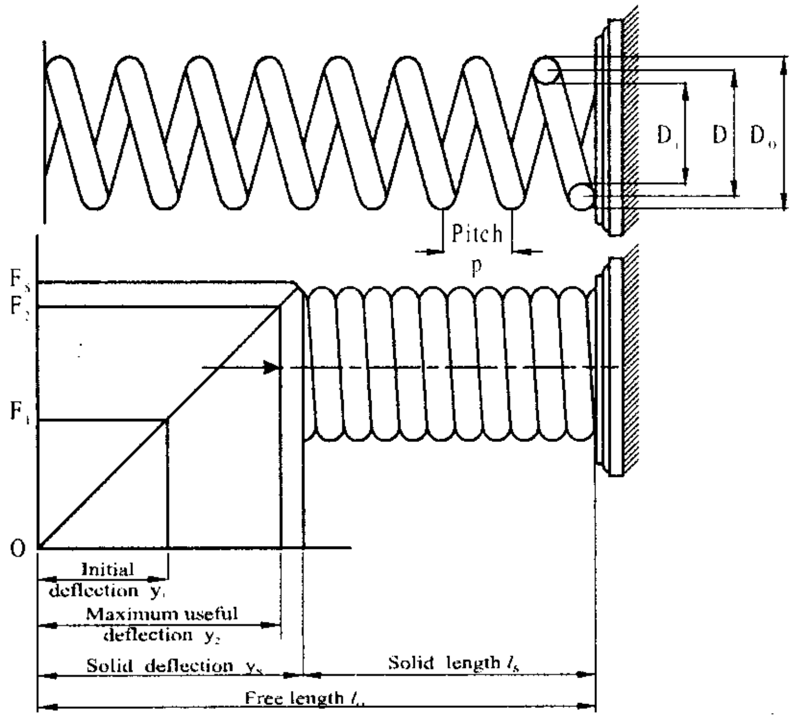


Fig. 3.3

l_o = Free length of spring

d = Diameter of spring wire

D = Mean diameter of coil

D_o = Outer diameter of coil

D_i = Inner diameter of coil

p = Pitch

F = Load on the spring or Axial force

i = Number of active coils

i' = Total number of coils

τ = Permissible shear stress or design shear stress

y = Deflection

G = Modulus of Rigidity

c = Spring Index

k = Curvature factor or Wahl's stress factor

k_o or F_o = Stiffness of spring or Rate of spring

a = Clearance → 25% of maximum deflection.

τ_y = Torsional yield shear strength (stress)

F.O.S = Factor of safely
F_1 = Minimum load
F_2 = Maximum load
y_2 = Maximum deflection
y' = Deflection for the load range
n = Number of additional coils → Table 20.6 (Old DDHB)
g = Acceleration due to gravity (Table 20.14 New DDHB)
V = Volume
m = Mass of the spring
ρ = Mass density of the spring
y_1 = Initial deflection or initial compression

3.11 DESIGN OF HELICAL SPRINGS

The design of a helical compression spring involves the following considerations :

1. Modes of loading - i.e., whether the spring is subjected to static or infrequently varying load or alternating load.
2. The force deflection characteristic requirement for the given application.
3. Is there any space restriction.
4. Required life for springs subjected to alternating loads.
5. Environmental conditions such as corrosive atmosphere and temperature.
6. Economy desired.

Considering these factors the designer select the material and specify the wire size, spring diameter, number of turns, spring rate, type of ends, free length and the surface condition.

A helical compression spring, that is too long compared to the mean coil diameter, acts as a flexible column and may buckle at a comparatively low axial force. Springs which cannot be designed buckle-proof must be guided in a sleeve or over an arbor. This is undesirable because the friction between the spring and the guide may damage the spring in the long run. It is therefore preferable, if possible, to divide the spring into buckle proof component springs separated by intermediate **platens** which are guided over an arbor or in a sleeve.

If, $$\frac{\text{Free length}}{\text{Mean coil diameter}} \leq 2.6 \text{ [Guide not necessary]}$$

$$\frac{\text{Free length}}{\text{Mean coil diameter}} > 2.6 \text{ [Guide required]}$$

3.12 Design procedure for helical compression spring of circular cross-section

*1. **Diameter of wire :***

Shear stress $$\tau = \frac{8FDk}{\pi d^3} \quad \text{---- 20.22}$$

Wahl's stress factork $= \frac{4c-1}{4c-4} + \frac{0.615}{c}$ **or 'k' from Fig. 20.5 (DDHB)**---- 20.23 (DDHB)

$c = \frac{D}{d}$ where d = diameter of spring wire

From Table 20.12 (Old DDHB) or Table 20.12 (New DDHB); select standard diameter for the spring wire.

2. ***Mean diameter of coil :***

Mean coil diameter $D = cd$

Outer diameter of coil $D_o = D + d$

Inner diameter of coil $D_i = D - d$

3. ***Number of coils :***

Axial Deflection $y = \frac{8FD^3.i}{d^4G}$ where i = Number of active turns or coils ---- 20.29(DDHB)

4. ***Free length :***

Free length $l_o \geq (i + n)\,d + y + a$ ---- 20.53 (DDHB)

Clearance a = 25% of maximum deflection ---- [Page 288 (Old DDHB Vol.I / Page 20.19 New DDHB Vol.I]

y = Maximum deflection.

Assume squared and ground end

∴ Number of additional coil n = 2

5. ***Stiffness or Rate of spring :***

Stiffness of spring $F_o = \frac{F}{y}$ ---- 20.30 (DDHB)

6. ***Pitch :***

Pitch $p = \frac{l_o - 2d}{i}$ ---- From Table 20.6 Old DDHB or Table 20.14 New DDHB

$= \frac{y_s}{i} + d$ 20.53 c (New DDHB)

Note :

1. For natural frequency of vibration of helical spring.

When one end is fixed :

$$f = \frac{1}{2\pi}\sqrt{\frac{2F_o}{m}}\ H_z \text{ or } \frac{1}{2\pi}\sqrt{\frac{2F_o g}{W}} \text{ or } \frac{1}{2\pi}\sqrt{\frac{2k_o g}{W}}$$ ---- 20.75 (DDHB)

When both ends are fixed :

$$f = \frac{1}{\pi}\sqrt{\frac{2F_o}{m}}\ H_z \text{ or } \frac{1}{\pi}\sqrt{\frac{2F_o g}{W}} \text{ or } \frac{1}{\pi}\sqrt{\frac{2k_o g}{W}}$$ ---- 20.76 (DDHB)

Mass m = Volume × Mass density where volume $V = \pi Di\left(\frac{1}{4}\pi d^2\right)$ ---- 20.33 (DDHB)

Weight W = Volume × Weight density = $\frac{\pi^2 d^2 D_i \gamma}{4}$ ---- 20.47g(New DDHB)

Density of steel ρ = 7.81 gm/cc = 7.81 × 10^{-6} kg/mm^3 ---- [Table 2.8 [Vol.I Old DDHB] Table 2.10 New DDHB Vol-1]

2. The factor of safety based on torsional yield strength (τ_y) is 1.5 for springs subjected to static forces.

$$\therefore \text{ Permissible shear stress } \tau = \frac{\tau_y}{1.5}$$

$$\text{Assuming } \sigma_{yt} = 0.75\,\sigma_{ut} \text{ and } \tau_y = 0.577\,\sigma_{yt}$$

$$\tau = \frac{(0.577)(0.75)\sigma_{ut}}{1.5} = 0.3\,\sigma_{ut}$$

∴ Permissible shear stress is approximately 30% of the ultimate tensile strength.

Or use fomulae 20.47a, 20.47b or 20.47c from New DDHB.

3.13 END CONDITION

While forming the ends of the helical compression spring, four methods are commonly used. They are plain ends, plain and ground ends, square ends and square and ground ends. Therefore, while calculating the number of active turns, the end turns should be subtracted from the total number of turns.

For plain end number of additional coils n = 0

For plain and ground end number additional coils n = $\frac{1}{2}$

For square end number of additional coils n = 2

For square and ground end number of additional coils n = 2

Example 3.1

Design a helical compression spring to support an axial load of 3000 N The deflection under load is limited to 60 mm. The spring index is 6. The spring is made of Chrome-vanadium steel and factor safety is equal to 2.

Data :

$$F = 3000\text{ N};\ y = 60\text{ mm};\ c = 6;\ FOS = 2$$

Solution :

From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB) for Chrome-vanadium steel

$$\tau_y = 690\text{ MPa} = 690\text{ N/mm}^2\ (0.69\text{ GPa})$$

$$G = 79340\text{ MPa} = 79340\text{ N/mm}^2\ (79.34\text{ GPa})$$

$$\therefore\ \tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345\text{ N/mm}^2$$

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3}$$ ---- 20.22 (DDHB)

$$\text{Wahl's stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$ ---- 20.23 (DDHB)

$$\text{Spring Index } c = \frac{D}{d}$$

$$6 = \frac{D}{d} \quad \therefore D = 6d$$

$$\therefore 345 = \frac{8\times3000\times6d\times1.2525}{\pi d^3}$$

$$\therefore d = 12.89\,\text{mm}$$

Select standard diameter of wire from Table 20.12 (Old DDHB)

$$\therefore \mathbf{d = 13\ mm}$$

2. Diameter of coil

$$c = \frac{D}{d}$$

$$6 = \frac{D}{13}$$

$\therefore$ Mean diameter of coil $\mathbf{D = 78\ mm}$

Outer diameter of coil $D_o = D+d = 78+13 = 91$ mm

Inner diameter of coil $D_i = D-d = 78-13 = 65$ mm

3. Number of coils or turns

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G}$$ ---- 20.29 (DDHB)

$$60 = \frac{8\times3000\times78^3\times i}{13^4\times79340}$$

$$i = 11.93$$

$\therefore$ Number of active turns $\mathbf{i = 12}$

4. Free length

$$l_o \geq (i+n)\,d + y + a$$

Clearance a = 25% of maximum deflection $= \frac{25}{100}\times 60 = 15$ mm

[Page 288 Vol.I Old DDHB or Page 20.19 New DDHB Vol.I]

Assume squared and ground end

$$\therefore n = 2$$

$\therefore$ Total number of turns $i' = i + n = 12 + 2 = 14$

$$\therefore l_o \geq (12+2)\,13+60+15 \geq 257\,\text{mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{257 - 2\times 13}{12} = 19.25 \text{ mm}$$ ---- From Table 20.6 (Old DDHB) Table 20.14 (New. DDHB)

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{3000}{60} = 50 \text{ N/mm}$$

Spring specifications

(i) Material Chrome vanadium steel
(ii) Wire diameter d = 13 mm
(iii) Mean coil diameter D = 78 mm
(iv) Free length l_o = 257 mm
(v) Total number of turns i' = 14
(vi) Style of ends - squared and ground
(vii) Pitch p = 19.25 mm
(viii) Rate of spring F_o = 50 N/mm

Example 3.2

A helical valve spring is to be designed for an operating load range of approximately 90 to 135 N. The deflection of the spring for the Load range is 7.5 mm. Assume a spring index of 10 and factor safety = 2. Design the spring.

Data :

Maximum load F_2 =135 N; Minimum load F_1 = 90 N; y' = 7.5 mm; c = 10; FOS = 2

Solution :

Assume chrome-vanadium alloy steel

∴ From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB)

$$\tau_y = 0.69 \text{ GPa} = 690 \text{ MPa} = 690 \text{ N/mm}^2$$

$$G = 79.34 \text{ GPa} = 79340 \text{ MPa} = 79340 \text{ N/mm}^2$$

$$\therefore \tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345 \text{ N/mm}^2$$

$$\text{Maximum deflection } y_2 = \frac{y' F_2}{F_2 - F_1}$$ ---- 20.31 (DDHB)

$$= \frac{7.5\times 135}{135-90} = 22.5 \text{ mm}$$

Design the spring for maximum load and maximum deflection

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3}$$ ---- 20.22 (DDHB)

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times 10-1}{4\times 10-4} + \frac{0.615}{10} = 1.1448$$ ---- 20.22 (DDHB)

$$\text{Spring index } c = \frac{D}{d} \quad \therefore D = cd = 10\,d$$

$$\therefore 345 = \frac{8 \times 135 \times 10d \times 1.1448}{\pi d^3}$$

$$d = 3.37 \text{ mm}$$

Select standard diameter of wire from Table 20.12 (Old DDHB)

$\therefore$ Diameter of wire **d = 3.4 mm**

2. Diameter of coil

Mean diameter of coil D = cd = 10 × 3.4 = 34 mm

Outer diameter of coil D_o = D + d = 34 + 3.4 = 37.4 mm

Inner diameter of coil D_i = D − d = 34 − 3.4 = 30.6 mm

3. Number of coils

$$y_2 = \frac{8F_2D^3i}{d^4G} \qquad \text{---- 20.29 (DDHB)}$$

$$22.5 = \frac{8 \times 135 \times 34^3 \times i}{3.4^4 \times 79340}$$

$$i = 5.62$$

$\therefore$ Number of active turns **i = 6**

4. Free length

$$l_o \geq (i + n)\,d + y + a \qquad \text{---- 20.53 (DDHB)}$$

$$a = 25\% \text{ of maximum deflection} = \frac{25}{100} \times 22.5 = 5.625 \text{ mm}$$

Assume squared and ground end

$\therefore$ Number of additional coil n = 2

y = maximum deflection y_2 = 22.5

$$\therefore l_o \geq (6 + 2)\,3.4 + 22.5 + 5.625$$

$$\geq 55.325 \text{ mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{55.325 - 2 \times 3.4}{6} = 8.0875 \text{ mm}$$

---- From Table 20.6 (Old DDHB) Table 20.14 (New DDHB)

6. Stiffness or Rate of spring

$$F_o = \frac{F_2}{y_2} = \frac{135}{22.5} = 6 \text{ N/mm} \qquad \text{---- 20.30 (DDHB)}$$

7. Total length of wire

$$l = \pi D i' \quad \text{where } i' = i + n = 6 + 2 = 8$$

$$= \pi \times 34 \times 8 = 854.513 \text{ mm}$$

Example 3.3

Design a valve spring for an automobile engine, when the valve is closed, the spring produces a force of 45 N and when it opens, produces a force of 55 N. The spring must fit over the valve bush which has an outside diameter of 20 mm and must go inside a space of 35 mm. The lift of the valve is 6 mm. The spring index is 12. The allowable stress may be taken as 0.33 GPa. Modulus of rigidity 80 GPa.

Data :

$F_1 = 45\text{ N}$ $\qquad c = 12$

$F_2 = 55\text{ N}$ $\qquad \tau = 0.33\text{ GPa} = 330\text{ MPa} = 330\text{ N/mm}^2$

$y' = 6\text{ mm}$ $\qquad G = 80\text{ GPa} = 80000\text{ MPa} = 80000\text{ N/mm}^2$

Solution :

$$\text{Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{55 \times 6}{55 - 45} = 33\text{ mm} \qquad \text{---- 20.31}$$

Design the spring for maximum deflection and maximum load

1. **Diameter of wire**

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \qquad \text{---- 20.22}$$

$$\text{Wahl's stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 12 - 1}{4 \times 12 - 4} + \frac{0.615}{12} = 1.119 \qquad \text{---- 20.23}$$

$$\text{Spring index } c = \frac{D}{d}$$

$$\therefore D = cd = 12\text{ d}$$

$$\therefore 330 = \frac{8 \times 55 \times 12d \times 1.119}{\pi d^3}$$

$$\therefore d = 2.387\text{ mm}$$

Select standard diameter of wire from Table 20.12 (Old DDHB)

$$\therefore \mathbf{d = 2.5\text{ mm}}$$

2. **Diameter of coil**

Mean diameter of coil $D = 12\text{ d} = 12 \times 2.5 = 30\text{ mm}$

Outer diameter $D_o = D + d = 30 + 2.5 = 32.5\text{ mm}$

Inner diameter of coil $D_i = D - d = 30 - 2.5 = 27.5\text{ mm}$

Check

$D_i = 27.5\text{ mm} > 20\text{ mm}$

$D_o = 32.5\text{ mm} < 35\text{ mm}$

$\therefore$ Safe i.e., possible

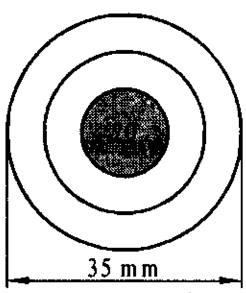


Fig. 3.4

3. **Number of coils**

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3 i}{d^4 G} \qquad \text{---- 20.29}$$

$$33 = \frac{8 \times 55 \times 33^3 \times i}{2.5^4 \times 80{,}000}$$

$$i = 8.68$$

$\therefore$ Number of active coils i $= \mathbf{9}$

4. Free length

$$l_o \geq (i + n)\, d + y + a \qquad \text{---- } 20.53$$

$$a = 25\% \text{ of maximum deflection} = \frac{25}{100} \times 33 = 8.25 \text{ mm}$$

$$y = \text{Maximum deflection } y_2 = 33 \text{ mm}$$

Assume squared and ground end

$\therefore$ Number of additional coils $n = 2$

$$\therefore\ l_o \geq (9 + 2)\, 2.5 + 33 + 8.25$$

$$l_o \geq 68.75 \text{ mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} \text{ ---- From Table 20.6 (Old DDHB) Table 20.14 (New DDHB)}$$

$$= \frac{68.75 - 2 \times 2.5}{9} = 7.083 \text{ mm}$$

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{F_2}{y_2} = \frac{55}{33} = 1.667 \text{ N/mm}$$

7. Total length of wire

$$l = \pi D i' \quad \text{where } i' = i + n = 9 + 2 = 11$$

$$= \pi \times 30 \times 11$$

$$= 1036.725 \text{ mm}$$

Example 3.4

Round wire cylindrical compression spring has an outside diameter of 75 mm. It is made of 12.5 mm diameter steel wire. The spring support an axil load of 5000 N, Determine

(i) Maximum shear stress.

(ii) Total deflection, if the spring has 8 coils with squared - ground end and is made of SAE 9260 steel.

(iii) Find also the pitch of coils and

(iv) The natural frequency of vibration of the spring if one end is at rest.

Data :

D_o = 75 mm; i' = 8; d = 12.5 mm; F = 5000 N; Material – SAE 9260

Solution :

From Table 20.14 Old DDHB or Table 20.10 New DDHB for SAE 9260

$$G = 79.34 \text{ GPa} = 79340 \text{ MPa} = 79340 \text{ N/mm}^2$$

1. Maximum shear stress

$$\textbf{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \qquad \text{---- 20.22}$$

$$D_o = D + d$$

$$75 = D + 12.5 \quad \therefore D = 62.5 \text{ mm}$$

$$c = \frac{D}{d} = \frac{62.5}{12.5} = 5$$

$$k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5} = 1.3105 \qquad \text{---- 20.23}$$

$$\therefore \tau = \frac{8\times5000\times62.5\times1.3105}{\pi\times12.5^3} = \mathbf{533.95\ N/mm^2}$$

2. Total deflection

For squared and ground end $n = 2$

$$\therefore i' = i + n$$

$$8 = i + 2$$

$$\therefore i = 6 = \text{number of active coils}$$

$$y = \frac{8FD^3i}{Gd^4} = \frac{8\times5000\times62.5^3\times6}{79340\times12.5^4} = \mathbf{30.25\ mm} \qquad 20.29$$

3. Pitch

$$p = \frac{l_o - 2d}{i} \text{ ---- From Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)}$$

$$l_o \geq (i+n)\,d + y + a$$

$$a = 25\% \text{ of maximum deflection}$$

$$= \frac{25}{100}\times30.25 = 7.5625 \text{ mm}$$

$$\therefore l_o \geq (6+2)\ 12.5 + 30.25 + 7.5625$$

$$\geq 137.8125 \text{ mm}$$

$$\therefore p = \frac{137.8125 - 2\times12.5}{6} = 18.8 \text{ mm}$$

4. Natural frequency

Natural frequency of vibration when one end is at rest

$$f = \frac{1}{2\pi}\sqrt{\frac{2F_o}{m}} = \frac{1}{2\pi}\sqrt{\frac{2F_og}{W}} \text{ Hz} \qquad \text{---- 20.75}$$

$$k_0 = F_0 = \frac{F}{y} = \frac{5000}{30.25} = 165.28925 \text{ N/mm} = 165289.25 \text{ N/m}$$

Mass m = Volume × density

$$\text{Volume } V = \pi Di\left(\frac{\pi d^2}{4}\right) \qquad \text{---- 20.33}$$

Mass density ρ = 7.81 gm/cc = 7.81×10^{-6} kg/mm³ for steel ---- From Table 2.8 (Old DDHB) or Table 2.10 (New DDHB)

Weight density $\gamma = 76.6\ kN/m^3 = 76.6 \times 10^{-6}\ N/mm^3$

$$\therefore m = \pi \times 62.5 \times 6 \times \pi \times \frac{12.5^2}{4} \times 7.81 \times 10^{-6} = 1.129\ kg$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2 \times 165289.25}{1.129}} = \mathbf{86.1\ Hz}$$

$$\text{or } W = \frac{\pi^2 d^2 D_i \gamma}{4} = \frac{\pi^2 \times 12.5^2 \times 62.5 \times 6 \times 76.6 \times 10^{-6}}{4} = 11.0744\ N$$

$$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2 \times 165289.25 \times 9.8066}{11.0744}} = 86.1\ Hz.$$

Example 3.5

The spring loaded safety valve for a boiler is required to blow off at a pressure of 1.3 MPa. The diameter of the valve is 65 mm and maximum lift of the valve is 17.5 mm. Design a suitable compression spring for the valve, assuming spring index to be 6 and providing initial compression of 30mm. Take τ = 0.45 GPa and G = 84 GPa.

Data :

P_1 = 1.3 MPa; Diameter of valve = 65 mm; y' = 17.5 mm c = 6; y_1 = 30 mm
τ = 0.45 GPa = 450 MPa = 450 N/mm²; G = 84 GPa = 84000 MPa = 84,000 N/mm²

Solution :

Maximum deflection $y_2 = y_1 + y' = 30 + 17.5 = 47.5$ mm ---- Refer Fig. 20.6

$$\text{Minimum Load } F_1 = P_1 \times \text{Area of valve} = 1.3 \times \frac{\pi}{4} \times 65^2 = 4313.8\ N$$

$$y_2 = \frac{F_2 y'}{F_2 - F_1} \quad \text{---- 20.31}$$

$$47.5 = \frac{F_2 \times 17.5}{F_2 - 4313.8}$$

$$F_2 - 4318.8 = 0.3684 F_2$$

$\therefore$ **Maximum Load F_2 = 6838.1 N**

Design the spring for maximum load and maximum deflection.

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525 \quad \text{---- 20.23}$$

$$c = \frac{D}{d}$$

$$\therefore D = cd = 6d$$

$$\therefore 450 = \frac{8 \times 6838.1 \times 6d \times 1.2525}{\pi d^3}$$

$\therefore d = 17.053 \text{ mm}$

Select standard diameter of wire from Table 20.12 (Old DDHB)

$\therefore \mathbf{d = 18 \text{ mm}}$

2. Diameter of coil

Mean diameter of coil $D = 6d = 6 \times 18 = 108 \text{ mm}$

Outer diameter $D_o = D + d = 108 + 18 = 126 \text{ mm}$

Inner diameter $D_i = D - d = 108 - 18 = 90 \text{ mm}$

3. Number of turns

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4.G} \qquad \text{----20.29}$$

$$47.5 = \frac{8 \times 6838.1 \times 108^3 \times i}{18^4 \times 84{,}000}$$

$$i = 6.078$$

$\therefore$ Number of active turns $\mathbf{i = 7}$

4. Free length

$$l_o \geq (i + n)\, d + y + a \qquad \text{----20.53}$$

y = Maximum deflection $y_2 = 47.5$ mm

$$a = \text{Clearance} = \frac{25}{100} \times \text{Max. deflection} = \frac{25}{100} \times 475 = 11.875 \text{ mm}$$

Assume squared and ground end

$\therefore$ number of additional coil $n = 2$

$$\therefore l_o \geq (7 + 2)\, 18 + 47.5 + 11.875$$

$$\geq 221.375 \text{ mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} \quad \text{--- From Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)}$$

$$= \frac{221.375 - 2 \times 18}{7} = 26.48 \text{ mm}$$

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{F_2}{y_2} = \frac{6838.1}{47.5} = 143.96 \text{ N/mm} \qquad \text{----20.30}$$

7. Total length :

$$l = \pi D i' \quad (\because i' = i + n = 7 + 2 = 9)$$

$$= \pi \times 108 \times 9 = 3053.63 \text{ mm}$$

Example 3.6

The valve spring of a gasoline engine is 40 mm long when the valve is open and 48 mm long when the valve is closed. The spring loads are 250 N when the valve is closed and 400 N when the valve is open. The inside diameter of the spring is not to be less than 25 mm and factor of safely is 2. Design the spring.

Data :

$$\mathbf{F_1 = 250\text{ N};\ F_2 = 400\text{ N};\ D_i = 25\text{ mm};\ y' = 48 - 40 = 8\text{ mm};\ FOS = 2}$$

Solution :

$$\text{Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{400 \times 8}{400 - 250} = 21.33 \text{ mm} \qquad \text{—20.31}$$

Design the spring for maximum load and maximum deflection

Assume Chrome vanadium alloy steel from Table 20.14 (Old DDHB) or Table 20.10 (New DDHB)

$$\tau_y = 0.69 \text{ GPa} = 690 \text{ MPa} = 690 \text{ N/mm}^2$$

$$G = 79.34 \text{ GPa} = 79.34 \times 10^3 \text{ MPa} = 79340 \text{ N/mm}^2$$

$$\therefore \tau = \frac{\tau_y}{FOS} = \frac{690}{2} = 345 \text{ N/mm}^2$$

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8F_2 Dk}{\pi d^3} \qquad \text{---- 20.22}$$

Assume $k = 1.25$ since 'c' is not given

$$D_i = D - d$$

$$\text{i.e., } D - d = 25$$

$$\therefore D = 25 + d$$

$$\therefore 345 = \frac{8 \times 400 \times (25 + d)1.25}{\pi d^3}$$

$$0.271d^3 = 25 + d$$

$$\text{i.e., } 0.271d^3 - d - 25 = 0$$

By hit and trial method $d = 4.791$ mm

Select standard diameter from Table 20.12 (Old DDHB). ∴ **d =5 mm**

2. Diameter of coil

Mean diameter of coil $D = 25 + d = 25 + 5 = 30$ mm

Outer diameter of coil $D_o = D + d = 30 + 5 = 35$ mm

Inner diameter of coil $D_i = 25$ mm

Check

$$\text{Spring index } c = \frac{D}{d} = \frac{30}{5} = 6$$

$$\therefore k = \frac{4c - 1}{4c - 4} + \frac{0.615}{c}$$

$$= \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

$$\therefore \tau_{cal} = \frac{8 \times 400 \times 30 \times 1.2525}{\pi \times 5^3}$$

$$\tau_{cal} = 306.2 \text{ N/mm}^2 < \tau_{allow} \text{ (i.e., } 345 \text{ N/mm}^2\text{)}$$

∴ Safe

3. Number of turns

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4G} \quad \text{---- 20.29}$$

$$\text{i.e., } 21.33 = \frac{8\times400\times30^3\times i}{5^4\times79340}$$

$$i = 12.24$$

$$\therefore \text{ Number of active turns } \mathbf{i} = \mathbf{13}$$

4. Free length

$$l_o \geq (i+n)\,d + y + a$$

$$a = 25\% \text{ of maximum deflection} = \frac{25}{100}\times21.33 = 5.3325$$

$$y = \text{maximum deflection } y_2 = 21.33$$

Assume squared and ground end $\therefore$ n = 2

$$\therefore l_o \geq (13+2)\,5 + 21.33 + 5.3325$$

$$\geq 101.6625 \text{ mm}$$

5. Pitch

$$p = \frac{l_0 - 2d}{i} = \frac{101.6625 - 2\times5}{13} = 7.05 \text{ mm}$$

---- From Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{F_2}{y_2} = \frac{400}{21.33} = 18.75 \text{ N/mm}$$

7. Total length:

$$l = \pi Di' = \pi\times30\times15 = 1413.72 \text{ mm} \quad (\because i' = i + n = 13 + 2 = 15)$$

Example 3.7

A spring controlled lever is shown in Fig. 3.5a. The spring is to be inserted with an initial compression to produce a force equal to 125 N between the right end of lever and the stop. When the maximum force at 'A' reaches a value of 200 N the end of the lever moves downward by 25 mm. Assume the spring index as 8 find (i) Spring rate (ii) Size of wire (iii) Outside diameter of spring (iv) Number of active coils (v) Free length and (vi) Pitch.

Solution :

$$W_1 = 125\text{N}$$

$$W_1 = 200\text{N}$$

Taking moments about '0' for minimum load

$$W_1\times400 = F_1\times200$$

$$125\times400 = F_1\times200$$

$$\therefore \mathbf{F_1 = 250\ N}$$

Taking moments about '0' for maximum load

$$W_2\times400 = F_2\times200$$

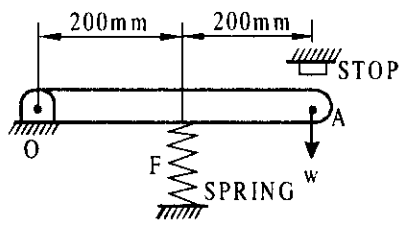


Fig. 3.5a

$$200 \times 400 = F_2 \times 200$$

$$\therefore \mathbf{F_2 = 400\ N}$$

By similar Δ^{ln} principle from Fig. 3.5b

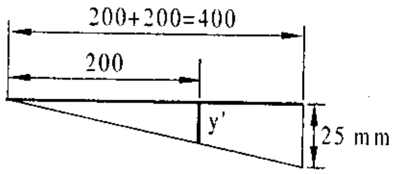


Fig. 3.5b

$$\frac{y'}{200} = \frac{25}{400}$$

$$\therefore y' = 12.5\,mm$$

$$\therefore \text{Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{400 \times 12.5}{400 - 250} = 33.33\ mm \quad \text{---- 20.31}$$

1. Spring rate

$$F_o = \frac{F_2}{y_2} = \frac{400}{33.33} = 12\,N/mm$$

2. Size of wire

Assume ASTM A 229 – 41 steel wire from Table 20.14 (Old DDHB) or Table 20.10 (New DDHB)

$$\tau_y = 0.55\ GPa = 550\ N/mm^2$$

$$G = 79.34\ GPa = 79340\ MPa = 79340\ N/mm^2$$

Assume factor of safety = 1.5

$$\therefore \tau = \frac{\tau_y}{FOS} = \frac{550}{1.5} = 366.67\ N/mm^2$$

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

$$\text{Spring index } c = \frac{D}{d}$$

$$\therefore D = cd = 8d$$

$$\therefore 366.67 = \frac{8 \times 400 \times 8d \times 1.184}{\pi d^3}$$

$$d = 5.13$$

Select standard diameter from Table 20.12 (Old DDHB)

$$\therefore \mathbf{d = 5.6\,mm}$$

3. Outside diameter of coil

$$\text{Mean diameter of coil } D = 8d = 8 \times 5.6 = 44.8\ mm$$

$$\text{Outside diameter of coil } D_o = D + d = 44.8 \times 5.6 = 50.4$$

4. Number of active coils

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3 i}{d^4 G} \quad \text{—20.29}$$

$$33.33 = \frac{8 \times 400 \times 44.8^3 \times i}{5.6^4 \times 79340}$$

$\therefore i = 9.038$

$\therefore$ Number active turns **i = 10**

5. Free length

$$l_o \geq (i + n)\,d + y + a$$

Clearance a = 25% of maximum deflection = 0.25 × 33.33 = 8.3325

Assume squared and ground end

$\therefore$ Number of additional coil n = 2

Total number of turns i' = i + n = 10 + 2 = 12

$$\therefore l_o \geq 12 \times 5.6 + 33.33 + 8.3325$$

$$\geq 108.8625 \text{ mm}$$

6. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{108.8625 - 2 \times 5.6}{10} = 9.77 \text{ mm}$$

---- From Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)

Spring specifications

(i) Material - ASTM A 229 – 41 steel wire

(ii) Wire diameter d = 5.6 mm

(iii) Mean coil diameter D = 44.8 mm

(iv) Free length l_o = 108.8625 mm

(v) Total number of turns i' = 12

(vi) Style of ends - squared and ground

(vii) Pitch p = 9.77 mm

Example 3.8

A closed helical spring is to have a stiffness of 1 N/mm, maximum load of 40 N and maximum shear stress of 130 N/mm². The solid length is 45 mm. Find the diameter of wire and number of coils required. Take G = 80 GPa = 80 × 10³ MPa

Data :

F_o = 1 N/mm; Solid length = 45 mm; F = 40 N; G = 80 GPa = 80000 N/mm²; τ = 130 N/mm²

Solution :

$$\text{Rate of spring } F_o = \frac{F}{y}$$

$$\text{i.e., } 1 = \frac{40}{y}$$

$\therefore$ Maximum deflection y = **40 mm**

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \qquad \text{---- 20.22}$$

Assume k = 1.25

$$\therefore 130 = \frac{8 \times 40 \times D \times 1.25}{\pi d^3}$$

$\therefore$ Mean coil diameter $D = 1.021\, d^3$ ---- (A)

$$\text{Maximum deflection } y = \frac{8FD^3 i}{d^4 G}$$

$$\text{i.e., } 40 = \frac{8 \times 40 \times \left(1.021 d^3\right)^3 (i)}{d^4 \times 80000}$$

$\therefore$ $\mathbf{d^5 i = 9395.56}$ ---- (B)

Solid length $i'd = 45$ mm

Assume plain end $\therefore$ $i' = i$

$\therefore$ $\mathbf{id = 45}$

$$\text{i.e., } \mathbf{i = \frac{45}{d}} \quad \text{---- (C)}$$

Sub in equation (B)

$$d^5 \times \frac{45}{d} = 9395.56$$

$\therefore$ $\mathbf{d = 3.8\ mm}$ = dia of wire

$D = 1.021 \times d^3 = 1.021 \times 3.8^3 = 56$ mm from A

= mean dia of coil

$$\text{From (C) } i = \frac{45}{d} = \frac{45}{3.8} = 11.84$$

$\therefore$ Number of active turns **i = 12**

Since plain end, number of additional coil n = 0

$\therefore$ Total number of coils $i' = 12$

Check :

$$\text{Spring index } c = \frac{D}{d} = \frac{56}{3.8} = 14.7368$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 14.7368 - 1}{4 \times 14.7368 - 4} + \frac{0.615}{14.7368} = 1.0963$$

$$\therefore \tau_{cal} = \frac{8FDk}{\pi d^3} = \frac{8 \times 40 \times 56 \times 1.0963}{\pi \times 3.8^3}$$

$= 113.96\ N/mm^2 < \tau_{allow}$ (i.e., $130\ N/mm^2$)

$\therefore$ Safe.

Example 3.9

Design a spring used in a recoil system so as to absorb 120 Nm of energy with a maximum force of 3000 N. Assume spring Index 8 and factor of safely is 2

Data :

$$\mathbf{U = 120\ Nm = 120 \times 10^3\ Nmm;\ c = 8;\ F = 3000\ N}$$

Solution :

Assume ASTM - A229 – 41 steel wire. From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB)

$$\tau_y = 0.55 \text{ GPa} = 550 \text{ MPa} = 550 \text{ N/mm}^2$$

$$G = 79.34 \text{ GPa} = 79340 \text{ MPa} = 79340 \text{ N/mm}^2$$

$$U = \frac{1}{2} Fy \quad \text{---- 20.32a}$$

$$120 \times 10^3 = \frac{1}{2} \times 3000 \times y$$

$$\mathbf{y} = \mathbf{80\ mm} = \text{Maximum deflection}$$

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times8-1}{4\times8-4} + \frac{0.615}{8} = 1.184$$

$$\text{Spring index } c = \frac{D}{d} \quad \therefore D = cd = \mathbf{8d}$$

$$\therefore 275 = \frac{8\times3000\times8d\times1.184}{\pi d^3}$$

$$d = 16.22 \text{ mm}$$

Select standard dia from Table 20.12 (Old DDHB)

$$\therefore \mathbf{d} = \mathbf{17\ mm}$$

2. Diameter of coil

Mean diameter of coil $D = 8d = 8 \times 17 = 136$ mm

Outer diameter of coil $D_o = D + d = 136 + 17 = 153$ mm

Inner diameter of coil $D_i = D - d = 136 - 17 = 119$ mm

3. Number of coils

$$\text{Maximum deflection } y = \frac{8FD^3 i}{d^4 G} \quad \text{---- 20.29}$$

$$\text{i.e., } 80 = \frac{8\times3000\times136^3\times i}{17^4\times79340} \quad \text{--- 20.29}$$

$$i = 8.78$$

$\therefore$ Number of active turns $\mathbf{i = 9}$

4. Free length

$$l_o \geq (i+n)\,d + y + a \quad \text{---- 20.53}$$

Assume squared and ground end

$\therefore$ Number of addition coils $n = 2$

Total number of coils $i' = i + n = 9 + 2 = 11$

Clearance a = 25% of maximum deflection = 0.25 × 80 = 20 mm

$$\therefore l_o \geq (11)(17) + 80 + 20$$

$$\geq 287 \text{ mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{287 - 2 \times 17}{9} = 28.11 \text{ mm}$$

---- From Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)

6. Rate of spring

$$F_o = \frac{F}{y} = \frac{3000}{80} = 37.5 \text{ N/mm}$$

7. Total length of wire

$$l = \pi D i' = \pi \times 136 \times 11 = 4699.82 \text{ mm}$$

Spring specifications

(i) Material - ASTM A 229 – 41 steel wire

(ii) Wire diameter d = 17 mm

(iii) Mean coil diameter D = 136 mm

(iv) Free length l_o = 287 mm

(v) Total number of turns i' = 11

(vi) Style of ends - squared and ground

(vii) Pitch p = 28.11 mm

(viii) Stiffness of spring F_o = 37.5 mm.

Example 3.10

A railway wagon weighing 50 kN and moving with a speed of 8 km/hr has to be stopped by four buffer springs in which the maximum compression allowed is 220 mm. Find the number of turns or coils in each spring of mean diameter 150 mm. The diameter of spring wire is 25 mm. Take G = 84 GPa. Also find the shear stress.

Data :

Weight of moving body W = 50 kN = 50,000 N

v = 8 km/hr = 8000/3600 = 2.2222 m/sec = 2222.222 km/hr

Number of springs = 4 **i = ?**

y = 220 mm **τ = ?**

D = 150 mm **d = 25 mm**

G = 84 GPa = 84,000 MPa = 84,000 N/mm²

Solution :

$$\text{Kinetic Energy} = \frac{1}{2} Mv^2 = \frac{1}{2}\frac{W}{g}v^2 = \frac{1}{2} \times \frac{50000}{9810} \times (2222.222)^2$$

$$= 12584790 \text{ Nmm}$$

$\therefore$ Energy stored by each spring $= \dfrac{\text{Total kinetic energy}}{\text{Number of springs}} = \dfrac{12584790}{4} = 3146197.5\,\text{Nmm}$

Also, Energy absorbed by each spring $= \dfrac{1}{2}Fy$ ---- 20.32

$$\text{i.e., } 3146197.5 = \frac{1}{2} \times F \times 220$$

$\therefore$ **F = 28601.8 N** = Axial force on each spring

1. Number of turns

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G} \quad \text{---- 20.29}$$

$$220 = \frac{8 \times 28601.8 \times 150^3 \times i}{25^4 \times 84{,}000} \quad \text{i.e., } i = 9.348$$

$\therefore$ Number of active turns **i = 10**

Assume squared and ground end

$\therefore$ Number of additional coil n = 2

$\therefore$ Total number of turns i' = i + n = 10 + 2 = 12

2. Shear stress

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Spring index } c = \frac{D}{d} = \frac{150}{25} = 6$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$

$$\therefore \tau = \frac{8 \times 28601.8 \times 150 \times 1.2525}{\pi \times 25^3} = 875.75\ \text{N/mm}^2$$

Example 3.11

A loaded narrow gauge car weighs 18 kN and moving at a velocity of 80 m/min is brought to rest by a buffer consists of two helical springs. In bringing the car to rest the spring undergoes a compression of 200 mm. The allowable shear stress is 0.3 GPa and the spring index is 8. Design a suitable spring. Take G = 84 GPa.

Data :

Weight of moving body W = 18 kN = 18,000 N

v = 80 m/min = 80/60 = 1.3333 m/sec = 1333.333 mm/sec

Number of springs = 2 ; y = 200 mm

τ = 0.3 GPa = 300 N/mm²

c = 8; G = 84 GPa = 84000 MPa = 84,000 N/mm²

Solution :

$$\text{Kinetic Energy} = \frac{1}{2} Mv^2 = \frac{1}{2}\frac{W}{g}.v^2$$

$$= \frac{1}{2} \times \frac{18000}{9810} \times 1333.333^2 = 1630988.8 \text{ Nmm}$$

$$\therefore \text{ Energy absorbed by each spring} = \frac{\text{Total kinetic energy}}{\text{Number of springs}} = \frac{1630988.8}{2} = 815494.4 \text{ Nmm}$$

$$\text{Also, Energy absorbed by each spring} = \frac{1}{2} Fy \quad \text{---- 20.32a}$$

$$\text{i.e., } 815494.4 = \frac{1}{2} \times F \times 200$$

$\therefore$ Axial force on each spring **F = 8154.944 N**

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times8-1}{4\times8-4} + \frac{0.615}{8} = 1.184$$

$$\text{Spring index } c = \frac{D}{d} \quad \therefore D = 8d$$

$$\therefore 300 = \frac{8\times8154.944\times8d\times1.184}{\pi d^3}$$

$$\text{i.e., } d = 25.6 \text{mm}$$

$\therefore$ Standard diameter of wire **d = 26 mm From Table 20.12 [OLD DDHB]**

2. Diameter of coil

Mean diameter of coil $D = 8d = 8 \times 26 = 208$ mm

Outer diameter of coil $D_o = D + d = 208 + 26 = 234$ mm

Inner diameter of coil $D_i = D - d = 208 - 26 = 182$ mm

3. Number of coils

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G} \quad \text{---- 20.29}$$

$$\text{i.e., } 200 = \frac{8\times8154.944\times208^3\times i}{26^4\times84000}$$

$$i = 13.076$$

$\therefore$ Number of active coils **i = 14**

4. Free length

$$l_o \geq (i+n)d + y + a \quad \text{---- 20.53}$$

Assume squared and ground end

$\therefore$ Number of addition coils n = 2

$\therefore$ Total number of turns $i' = i + n = 14 + 2 = 16$

clearance a = 25% of deflection $= 0.25 \times 200 = 50$ mm

$$l_o \geq 16\times26+200+50$$

$$\geq 666 \text{mm}$$

5. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{666 - 2\times 26}{14} = 43.86 \text{ mm} \text{ ---- From table 20.6}$$

6. Rate of spring

$$F_o = \frac{F}{y} = \frac{8154.944}{200} = 40.775 \text{ N/mm}$$

7. Total length

$$l = \pi D i' = \pi \times 208 \times 16 = 10455.22 \text{ mm}$$

Spring specifications

(i) Wire diameter d = 26 mm

(ii) Mean coil diameter D = 208 mm

(iii) Free length l_o = 666 mm

(iv) Total number of turns i' = 16

(v) Style of ends - squared and ground

(vi) Pitch p = 43.86 mm

(vii) Rate of spring F_o = 40.775 N/mm.

Example 3.12

A load of 2000 N is dropped axially on a closed coiled helical spring from a height of 250 mm. The spring has 20 effective turns, and it is made of 25 mm diameter wire. Find the maximum shear stress produced in the spring and the amount of compression produced. Take c = 8 and G = 84 GPa

Data :

Weight of falling body W = 2000 N; h =250 mm; G = 84 GPa = 84 × 10³ N/mm²

i = 20; d = 25 mm; c = 8

Solution :

$$\text{Potential Energy} = (h + y)\, W = (250 + y)\, 2000$$

$$= 5 \times 10^5 + 2000\, y = \text{Energy stored by the spring}$$

Also energy stored by the spring = $\frac{1}{2}$ Fy ---- 20.32

$$\therefore \frac{1}{2} Fy = 5 \times 10^5 + 2000\, y \quad (\because \text{ Number of springs} = 1)$$

$$\text{i.e., } \mathbf{Fy = 10^6 + 4000\, y} \qquad \text{---- (A)}$$

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G} = \frac{8 \times F \times 200^3 \times 20}{25^4 \times 84.000} \quad [\because c = \frac{D}{d}; 8 = \frac{D}{25}; \therefore D = 200\text{mm}]$$

$$\therefore \mathbf{y = 0.039\, F} \qquad \text{----- (B)}$$

Sub in equation (A)

$$0.039 F^2 = 10^6 + 4000 \times 0.039 F$$

$$\text{i.e. } F^2 = 25.635 \times 10^6 + 4000 F$$

$$\therefore\ F^2 - 4000F - 25.635 \times 10^6 = 0$$

$$\therefore\ F = \frac{+4000 \pm \sqrt{4000^2 + 4 \times 1 \times 25.635 \times 10^6}}{2 \times 1}$$

taking positive value

Axial load on the spring **F = 7443.8 N**

1. Shear stress

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \qquad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 8 - 1}{4 \times 8 - 4} + \frac{0.615}{8} = 1.184$$

$$\therefore\ \tau = \frac{8 \times 7443.8 \times 200 \times 1.184}{\pi \times 25^3} = \mathbf{287.274\ N/mm^2}$$

2. Compression or deflection

From (B) $y = 0.039\,F = 0.039 \times 7443.8 = \mathbf{290.3\ mm}$

Example 3.13

Design a spring for an elevator shaft at the bottom of which 8 identical springs are set in parallel to absorb the shock of the elevator in case of failure. The weight of elevator is 60 kN and the counter weight of elevator is 20 kN. The elevator has a free fall of 1.5 m from rest. The spring is made of 25 mm diameter rod. Determine the maximum stress in each spring, if the spring index is 6. Each spring has 15 active turns. Take G = 84 GPa

Data :

Number of springs = 8; Weight of elevator = 60 kN = 60,000 N

Weight of counter weight = 20 kN = 20,000 N; h = 1.5 m = 1500 mm; d = 25 mm

c = 6; i = 15; G = 84 GPa = 84000 MPa = 84000 N/mm²

Solution :

Weight of falling body W = Weight of elevator– Weight of counter Wt = 60,000 – 20,000 = 40,000 N

$$\text{Spring index } c = \frac{D}{d} \quad \therefore\ D = cd = 6 \times 25 = 150\ \text{mm}$$

$$\text{Total potential energy} = W(h + y) = 40{,}000\,[1500 + y] = [60 \times 10^6 + 4 \times 10^4\,y]$$

$$\therefore\ \text{Energy stored by each spring} = \frac{\text{Total P.E}}{\text{Number of springs}}$$

$$= \frac{60 \times 10^6 + 4 \times 10^4 . y}{8} = 7.5 \times 10^6 + 5 \times 10^3\,y$$

$$\text{Also, energy stored by each spring} = \frac{1}{2}Fy$$

$$\therefore\ 7.5 \times 10^6 + 5 \times 10^3\,y = \frac{1}{2}Fy$$

i.e., $Fy = 15\times10^6+10\times10^3 y$ ---- (A)

$$\text{Deflection } y = \frac{8FD^3i}{d^4G} = \frac{8\times F\times150^3\times15}{25^4\times84{,}000} = \mathbf{0.012343\ F}$$ ---- 20.29 (B)

Sub in equation (A)

$$0.012343\ F^2 = 15\times10^6 + 10\times10^3\times0.012343\ F$$

$$\text{i.e., } F^2 = 1215.264\times10^6 + 10^4\ F$$

$$\therefore\ F^2 - 10^4\ F - 1215.264\times10^6 = 0$$

$$\therefore\ F = \frac{+10^4 \pm \sqrt{(10^4)^2 + 4\times1\times1215.264\times10^6}}{2\times1}$$

Taking positive sign

Axial force on each spring F = 40217.4 N

i. Deflection

From (B)

$$\text{Deflection } y = 0.012343\ F = 0.012343\times40217.4 = \mathbf{496.4\ mm}$$

ii. Shear stress

$$\tau = \frac{8FDk}{\pi d^3}$$ ---- 20.22

$$\text{Wahl's stress factor } k = \frac{4c-1}{4c-4}+\frac{0.615}{c} = \frac{4\times6-1}{4\times6-4}+\frac{.615}{6} = 1.2525$$

$$\therefore\ \tau = \frac{8\times40217.4\times150\times12525}{\pi\times25^3} = \mathbf{1231.4\ N/mm^2}$$

= Maximum allowable shear stress

iii. Free length

$$l_o \geq (i+n)\,d + y + a$$ ---- 20.53

Assume squared and ground end

$\therefore$ Number of additional coil n = 2

Total number of turns i' = i + n = 15 + 2 = 17

clearance a = 25% of maximum deflection = 0.25 × 496.4 = 124.1

$$\therefore\ l_o \geq 17\times25 + 496.4 + 124.1$$

$$\geq 1045.5\ mm$$

iv. Pitch

$$p = \frac{l_o - 2d}{i} = \frac{1045.5-2\times25}{15} = 66.37\ mm$$

---- Table 20.6 (Old DDHB) or Table 20.14 (New DDHB)

v. Rate of spring

$$F_o = \frac{F}{y} = \frac{40217.4}{496.4} = 81.018\ N/mm$$

vi. Total length of wire

$$l = \pi Di' = \pi \times 150 \times 17 = 8011.06 \text{ mm}$$

Spring Specifications

i) Wire diameter d = 25 mm
ii) Mean coil diameter D = 150 mm
iii) Free length l_0 = 1045.5 mm
iv) Total number of turns i' = 17
v) Style of ends – squared and ground
vi) Pitch p = 66.37 mm
vii) Spring stiffness or Rate of spring F_0 = 81.018 N/mm

Example 3.14

A single plate friction clutch transmits 20 kW at 1000 rpm. There are 2 pairs of friction surfaces having a mean radius of 150 mm. The axial pressure is provided by six springs. If the springs are compressed by 5 mm during declutching, design the spring. Take c = 6, τ = 0.42 GPa, G = 80 GPa and μ = 0.3.

Data :

$$N = 20 \text{ kW} \qquad c = 6$$
$$n = 1000 \text{ rpm} \qquad \tau = 0.42 \text{ GPa} = 420 \text{ MPa}$$
$$i = \text{number of active surfaces} = 2$$
$$R_m = 150 \text{ mm} \quad \therefore D_m = 300 \text{ mm}$$
$$\text{Number of springs} = 6$$
$$y = 5 \text{ mm}$$

Solution :

Clutch

$$\text{Torque } M_t = 9550 \times 1000 \times \frac{N}{n} = 9550 \times 1000 \times \frac{20}{1000} = 191000 \text{ Nmm}$$

$$\text{Also, } M_t = \frac{1}{2} \mu F_a D_m i \text{ for disc dutch} \qquad \text{---- 19.84}$$

$$\therefore 191000 = \frac{1}{2} \times 0.3 \times F_a \times 300 \times 2$$

$$F_a = 2122.22 \text{ N} = \text{Axial force}$$

$$\therefore \text{Axial force on each spring } F = \frac{F_a}{\text{Number of springs}} = \frac{2122.22}{6} = \mathbf{353.7\ N}$$

1. Diameter of the wire

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \qquad \text{---- 20.22}$$

$$\text{Stres factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$

$$\text{Spring index } c = \frac{D}{d} = 6 \;\therefore\; D = 6d$$

$$\therefore\; 420 = \frac{8 \times 353.7 \times 6d \times 1.2525}{\pi d^3}$$

$$\mathbf{d} = \mathbf{4\ mm} = \text{diameter of wire}$$

2. **Diameter of coil**

Mean diameter of coil $D = 6d = 6 \times 4 = 24$ mm

Outer diameter of coil $D_o = D + d = 24 + 4 = 28$ mm

Inner diameter of coil $D_i = D - d = 24 - 4 = 20$ mm

3. **Number of turns**

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G} \qquad \text{---- 20.29}$$

$$\text{i.e., } 5 = \frac{8 \times 353.7 \times 24^3 \times i}{4^4 \times 80000}$$

$$\therefore\; i = 2.61$$

$\therefore$ Number of active terms or coils $i = 3$

4. **Free length**
5. **Pitch**
6. **Stiffness**
7. **Total length of wire and specification of spring wire are as calculated in earlier Examples.**

Example 3.15

A single plate friction clutch is to be designed for a vehicle. Both sides of the plate are to be effective. The clutch transmits 30 kW at a speed of 3000 rpm and should cater for an overload of 20%. The intensity of pressure on the friction surfaces should not exceed 0.085 N/mm² and the surface speed at the mean radius should be limited to 2300 m/min. The outside diameter may be assumed as 1.3 times inside diameter and μ = 0.3. If the axial thrust is to be provided by six springs of about 25 mm coil diameter design the springs selecting the wire from the following gauges. Safe shear stress is limited to 0.42 GPa and modulus of rigidity 84 GPa.

SWG	4	5	6	7	8	9	10	11	12
Dia	5.893	5.385	4.877	4.47	4.064	3.658	3.251	2.546	2.642

Data :

Number of active surfaces for clutch i = 2; N =30 kW; Number of springs = 6

n = 3000 rpm; D = 25 mm; Over load = 20%; G = 84 GPa = 84000 MPa

p = 0.085 N/mm²; τ = 0.42 GPa = 420 MPa; v = 2300 N/min

$\mathbf{D_2 = 1.3\,D_1;\ \mu = 0.3}$

Solution :

Clutch

$$\text{Torque } M_t = 9550 \times 1000 \times \frac{N}{n} \times 1.2 \quad (\because \; 20\% \text{ overload})$$

$$= 9550 \times 1000 \times \frac{30}{3000} \times 1.2 = 114600 \text{ Nmm}$$

$$\text{For disc clutch, } M_t = \frac{1}{2} \mu F_a D_m i \qquad \text{---- } 19.84$$

$$\text{Assume uniform wear } \therefore \; D_m = \frac{1}{2}(D_2 + D_1) = \frac{1}{2}(1.3\,D_1 + D_1) = 1.15\,D_1$$

$$F_a = \frac{1}{2} \pi p D_1 (D_2 - D_1) \qquad \text{---- } 19.83$$

$$= \frac{1}{2} \pi \times 0.085\, D_1 (1.3 D_1 - D_1) = 0.04\, D_1^2$$

$$\therefore \; 114600 = \frac{1}{2} \times 0.3 \times 0.04\, D_1^2 \times 1.15\, D_1 \times 2$$

Inner diameter of friction surface $D_1 = 202.5$ mm

∴ Outer diameter of friction surface $D_2 = 263.25$ mm

Check

$$v_m = \frac{\pi D_m n}{60000} = \frac{\pi \times 1.15 \times 202.5 \times 3000}{60000}$$

$$= 36.58 \text{ m/sec} = 2195 \text{ m/min} < 2300 \text{ m/min}$$

∴ Safe

$$\text{Axial Force } F_a = 0.04 \times D_1^2 = 0.04 \times 202.5^2 = 1640.25 \text{ N}$$

$$\therefore \; \text{Load on each spring} = \frac{F_a}{\text{Number of springs}} = \frac{1640.25}{6} = 273.375 \text{ N} = F$$

= Axial load on each spring

Spring :

i) **Diameter of wire**

$$\text{Shear stress } \tau = \frac{8FDk}{\pi d^3} \qquad \text{----} 20.22$$

Assume $k = 1.25$

$$420 = \frac{8 \times 273.375 \times 25 \times 1.25}{\pi \times d^3}$$

$$\therefore \; d = 3.72 \text{ mm}$$

From the given table select **SWG 8** wire

∴ Diameter of wire **d = 4.064 mm**

Check

$$\text{Spring index } c = \frac{D}{d} = \frac{25}{4.064} = 6.1515$$

$$\therefore \text{ Stress factor } k = \frac{4\times 6.1515-1}{4\times 6.1515-4}+\frac{0.615}{6.1515} = 1.2456$$

$$\therefore \tau_{cal} = \frac{8\times 273.375\times 25\times 1.2456}{\pi\times 4.064^3}$$

i.e., $\tau_{cal} = 322.96\text{ N/mm}^2 < \tau_{allow}$ (i.e., 420 N/mm²) $\therefore$ safe.

ii) Number of Coils

$$\text{Deflection } y = \frac{8FD^3 i}{d^4 G} \qquad \text{---- 20.29}$$

Assume allowable compression, y = 5 mm

$$\therefore 5 = \frac{8\times 273.375\times 25^3\times i}{4.064^4\times 84000}$$

$$i = 3.35$$

$\therefore$ Number of active turns **i = 4**

iii) Free length

iv) Pitch

v) Stiffness

vi) Total lengths and specification of spring wire are as calculated in earlier Examples.

Example 3.16

Design the spring for the Hartnell type spring loaded governor for the following particulars. Mass of each ball = 2.97 kg, length of vertical or ball arm is 150 mm, length of sleeve or horizontal arm is 112.5 mm. The governor is begin to lift at a speed of 240 rpm and the maximum speed is 7.5% higher than that. The maximum radius of rotation is 150 mm and the minimum radius of rotation is 100 mm. The allowable stress on the spring material is 0.42 GPa and modulus of rigidity is 84 GPa. Take c = 8.

Data :

For Governor	For spring
m = 2.97 kg; r_1 = 100 mm = 0.1 meter	**τ = 0.42 GPa = 420 MPa**
a = 150 mm = 0.15 m; r_2 = 150 mm = 0.15 meter;	**G = 84 GPa = 84000 MPa; c = 8**
b = 112.5 mm = 0.1125 m	

$$N_1 = 240 \text{ rpm};\ N_2 = 240 + \frac{7.5}{100}\times 240 = 258 \text{ rpm}$$

Solution :

$$F_{C1} = m\omega_1^2 r_1 = m\left(\frac{2\pi N_1}{60}\right)^2 r_1 = 2.97\times\left(\frac{2\pi 240}{60}\right)^2\times 0.1 = 187.6\text{ N}$$

$$F_{C2} = m\omega_2^2 r_2 = m\left(\frac{2\pi N_2}{60}\right)^2 . r_2 = 2.97 \times \left(\frac{2\pi 258}{60}\right)^2 (0.15) = 325.2\text{ N}$$

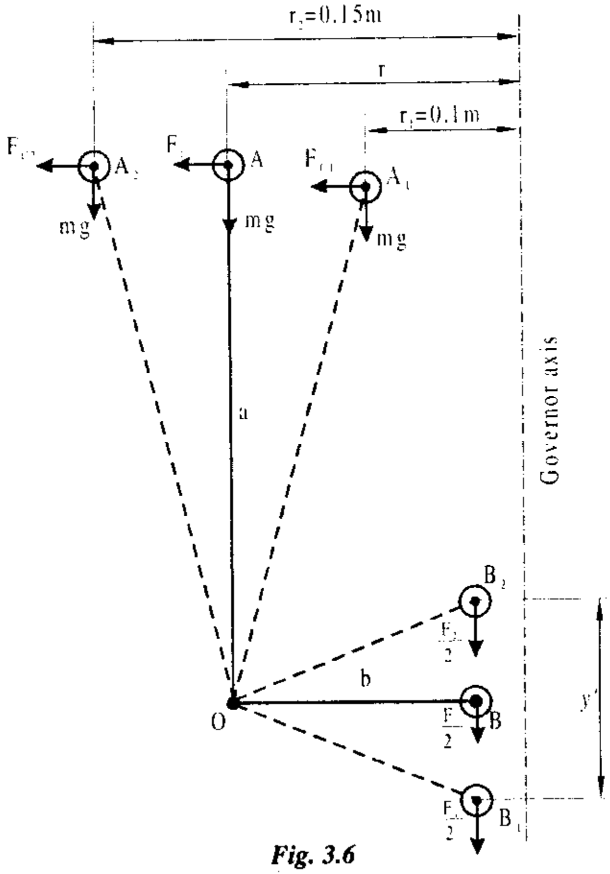


Fig. 3.6

Taking moments about 'O' for maximum radius neglecting the obliquity of arms and mass of ball

$$F_{C2} \times a = \frac{F_2}{2} \times b$$

$$325.2 \times 0.15 = \frac{F_2}{2} \times 0.1125$$

$$\mathbf{F_2 = 867.2\text{ N}}$$

Taking moments about 'O' for minimum radius

$$F_{C1} \times a = \frac{F_1}{2} \times b$$

$$187.6 \times 0.15 = \frac{F_1}{2} \times 0.1125$$

$$\mathbf{F_1 = 500.27\text{ N}}$$

$\Delta^{le}OB_1B_2$ and OA_1A_2 are similar

$$\therefore \frac{B_1B_2}{A_1A_2} = \frac{b}{a}$$

$$\frac{y'}{0.15-0.1} = \frac{0.1125}{0.15}$$

$\therefore$ **y' = 0.0375 meter = 37.5 mm** = Deflection for the load range

$$\therefore \text{ Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{867.2 \times 37.5}{867.2 - 500.27} = 88.627 \text{ mm} \quad \text{---- 20.31}$$

Design the spring for maximum load and maximum deflection.

1) Diameter of Wire

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} \quad \text{---- 20.23}$$

$$= \frac{4\times8-1}{4\times8-4} + \frac{0.615}{8} = 1.184; \quad c = \frac{D}{d} \quad D = cd = 8d$$

$$\therefore 420 = \frac{8 \times 867.2 \times 8d \times 1.184}{\pi \times d^3}$$

$$d = 7.06 \text{ mm}$$

Select std diameter from Table 20.12 (Old DDHB)

$\therefore$ Diameter of wire **d = 7.5 mm**

2) Diameter of coil

Mean diameter $D = 8d = 8 \times 7.5 = 60$ mm

Outer diameter $D_o = D + d = 60 + 7.5 = 67.5$ mm

Inner diameter $D_i = D - d = 60 - 7.5 = 52.5$ mm

3) Number of turn

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4G} \quad \text{---- 20.29}$$

$$\text{i.e., } 88.627 = \frac{8 \times 867.2 \times 60^3 \times i}{7.5^4 \times 84000}$$

$$\therefore i = 15.72$$

$\therefore$ Number of active turns **i = 16**

4) Free length

5) Pitch

6) Rate of spring

7) Total length of wire and specification of spring wire are as calculated in earlier Examples.

Example 3.17

Spring loaded Governor with dimensions shown in Fig. 3.7 a needs a spring force of 190 N to permit proper speed of balls of 125 mm radius. Assuming limiting position to be shown that in Fig. 3.7 b with balls at 175 mm radius corresponding the spring force is 650 N. Take G = 84 GPa, τ = 0.35 GPa and c = 8. Determine (i) Rate of spring (ii) Wire diameter and coil diameter and (iii) number of coils.

Data :

Governor	Spring
$F_1 = 190$ N	$\tau = 0.35$ GPa = 350 MPa
$F_2 = 650$ N	$G = 84$ GPa = 84000 MPa
$r_1 = 125$ mm	$c = 8$
$r_2 = 175$ mm	

Solution :

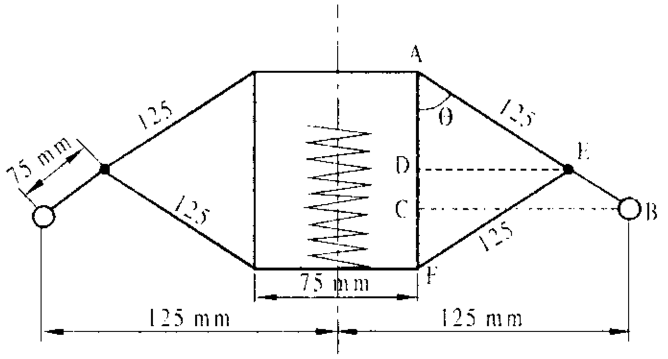


Fig. 3.7 a

From Fig. 3.7 a

$$CB = 125 - \frac{75}{2} = 87.5 \text{ mm}$$

From Δ[le] ACB

$$\sin\theta = \frac{CB}{AB} = \frac{87.5}{125+75} = \frac{87.5}{200}$$

$$\therefore\ \theta = \mathbf{25.944°}$$

From Δ[le] ADE

$$\cos\theta = \frac{AD}{AE} \qquad \sin\theta = \frac{DE}{AE}$$

$$\cos 25.944 = \frac{AD}{125} \qquad \sin 25.944 = \frac{DE}{125}$$

$$\therefore\ AD = 112.4 \text{ mm} \qquad \therefore\ DE = 54.686 \text{ mm}$$

From Δ[le] DEF

$$DF = \sqrt{FE^2 - DE^2} = \sqrt{125^2 - 54.686^2} = 112.4$$

$$\therefore\ AF = AD + DF = 112.4 + 112.4 = \mathbf{224.8\ mm}$$

For the maximum radius from Fig. 3.7b

$$C_1B_1 = 175 - \frac{75}{2} = 137.5 \text{ mm}$$

For the maximum radius

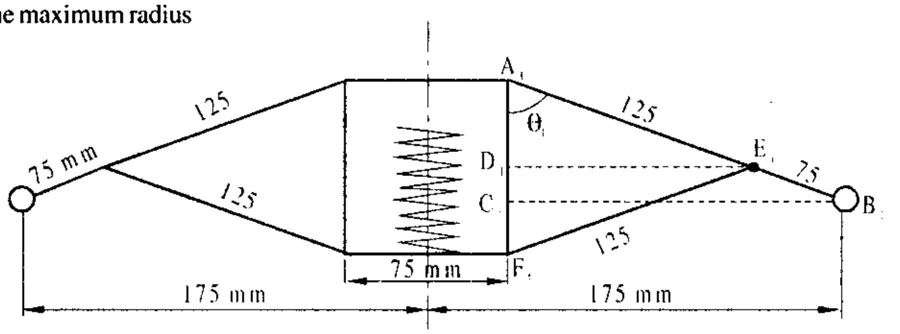


Fig. 3.7 b

From Δ^{le} $A_1C_1B_1$

$$\sin\theta_1 = \frac{C_1B_1}{A_1B_1} = \frac{137.5}{125+75} = \frac{137.5}{200}$$

$$\therefore \ \boldsymbol{\theta_1 = 43.43°}$$

From Δ^{le} $A_1\ D_1\ E_1$

$\cos\theta_1 = \frac{A_1D_1}{A_1E_1}$	$\sin\theta_1 = \frac{D_1E_1}{A_1E_1}$
$\cos 43.43 = \frac{A_1D_1}{125}$	$\sin 43.43 = \frac{D_1E_1}{125}$
$\therefore$ **A_1D_1 = 90.776 mm**	$\therefore$ **D_1E_1 = 85.93 mm**

From Δ^{le} $D_1E_1F_1$

$$D_1F_1 = \sqrt{F_1E_1^2 - D_1E_1^2} = \sqrt{125^2 - 85.93^2} = 90.776 \text{ mm}$$

$$\therefore \ A_1F_1 = A_1D_1 + D_1F_1 = 90.776 + 90.776 = 181.552 \text{ mm}$$

$$\therefore \ y' = AF - A_1F_1 = 224.8 - 181.552$$

$$= 43.248 \text{ mm} = \text{deflection for the load range}$$

$$\therefore \ \text{Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{43.248 \times 650}{650 - 190} = \mathbf{61.11 \text{ mm}} \qquad \text{----20.31}$$

Design the spring for maximum load and maximum deflection

1) Rate of spring

$$F_0 = \frac{F_2}{y_2} = \frac{650}{61.11} = 10.636 \text{ N/mm}$$

2) Diameter of wire and coil diameter

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \qquad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times8-1}{4\times8-4} + \frac{0.615}{8} = 1.184 \qquad \text{----20.23}$$

$$\text{Spring index } c = \frac{D}{d} \qquad \therefore\ D = cd = 8d$$

$$\text{i.e., } 350 = \frac{8\times650\times8d\times1.184}{\pi d^3}$$

$$\therefore\ d = 6.69 \text{ mm}$$

Select Std diameter from Table 20.12 (Old DDHB)

d = 7 mm = diameter of wire

Mean coil diameter $D = 8d = 8 \times 7 = 56$ mm

Outer diameter of coil $D_o = D + d = 56 + 7 = 63$ mm

Inner diameter of coil $D_i = D - d = 56 - 7 = 49$ mm

3) Number of coils

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4.G} \qquad \text{---- 20.29}$$

$$61.11 = \frac{8\times650\times56^3\times i}{7^4\times84000}$$

$$\therefore\ i = 13.49$$

$\therefore$ Number of active turns **i = 14**

Example 3.18

In a free rolling conveyor shown in Fig. 3.8 a crate loaded with non-frogile material 300 N reaches station A at a velocity of 3 m/sec and is normally unloaded at section B. In case it gets away from B, a *pair of helical* buffer spring at C are expected to arrest the motion of crate. It is suggested that the springs may be set up with an initial compression of about 50 mm and that compression of spring due to impact of load may be limited to 150 mm. Design the spring. Take G = 80 GPa, τ = 0.8 GPa and c = 6.

Data :

W = 300 N; v = 3 m/sec = 3000 mm/sec; y_1 = 50 mm; y' = 150 mm

G = 80 GPa = 80000 N/mm²; τ = 0.8 GPa = 0.8 × 1000 = 800 MPa; c = 6

Solution :

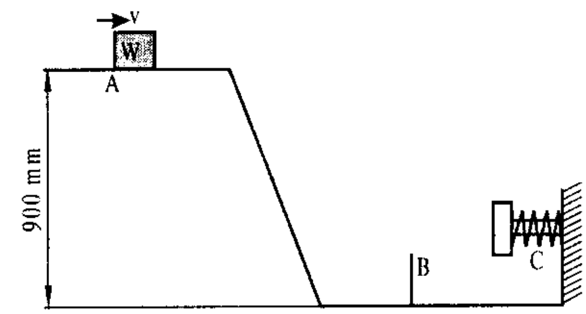


Fig. 3.8

$$KE = \frac{1}{2}Mv^2 = \frac{1}{2}\frac{W}{g}v^2 = \frac{1}{2}\times\frac{300}{9810}\times(3000)^2 = 137614.679\,Nmm$$

$$P.E. = Mgh = Wh = 300\times900 = 27\times10^4\,Nmm$$

$$\therefore \text{ Total energy} = KE + PE = 137614.679 + 27\times10^4 = 407614.679\,Nmm$$

$$\therefore \text{ Energy stored on each spring} = \frac{407614.679}{2\,(\text{Number of springs})} = 203807.339\text{ Nmm}$$

Also, Energy stored by each spring = $\frac{1}{2}$ Fy ---- 20.32a

$$\text{i.e., } U = \text{Average force} \times \text{deflection} = \frac{1}{2}(F_1 + F_2)\,y'$$

$$F_0 = \frac{F_1}{y_1} = \frac{F_2}{y_2} = \frac{F}{y'}$$

$$\therefore F_1 = F_0 y_1 \qquad y_2 = y_1 + y' = 50 + 150 = 200$$

$$F_2 = F_0 y_2 \qquad = \text{Maximum deflection}$$

$$\therefore U = \frac{1}{2}(F_0 y_1 + F_0 y_2)\,y' = \frac{1}{2}F_0 (y_1 + y_2)\,y'$$

$$\therefore 203807.339 = \frac{1}{2}F_0\,(50 + 200)\,150$$

$$\therefore F_0 = 10.8697\text{ N/mm} = \text{Rate of spring}$$

$$\therefore F_1 = F_0 y_1 = 10.8697 \times 50 = 543.5\text{ N}$$

$$F_2 = F_0 y_2 = 10.8697 \times 200 = 2174\text{ N}$$

$\therefore$ Minimum load F_1 = 543.5 N

Maximum load F_2 = 2174 N

Maximum deflection y_2 = 200 mm

Design the spring for maximum load and maximum deflection.

1) Diameter of wire

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \qquad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$

$$\text{Spring index } c = \frac{D}{d} \quad \therefore D = cd = 6d$$

$$\therefore 800 = \frac{8\times2174\times6d\times1.2525}{\pi d^3};\ d = 7.21\text{ mm}$$

Select Std diameter from Table 20.12 (Old DDHB) $\therefore$ **d = 7.5 mm** = diameter of wire

2) Diameter of coil

Mean diameter of coil $D = 6d = 6\times7.5 = 45$ mm

Outer diameter of coil $D_0 = D + d = 45 + 7.5 = 52.5$ mm

Inner diameter of coil $D_i = D - d = 45 - 7.5 = 37.5$ mm.

3) Number of turns

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4G} \quad \text{---- 20.29}$$

$$\text{i.e., } 200 = \frac{8\times 2174\times 45^3\times i}{7.5^4\times 80000}$$

$$i = 31.94.$$

$$\therefore \text{ Number of active turns } i = \mathbf{32}$$

4) Free length.

5) Pitch

6) Rate of spring

7) Total length of wire and specification of spring wire are as calculated in earlier Examples.

3.14 CONCENTRIC OR COMPOSITE SPRINGS

A concentric spring is used for one of the following purposes:

i) To obtain greater spring force within a given space.

ii) To insure the operation of a mechanism in the event of failure of one of the springs.

Assume both the springs are made of same material, then maximum shear stress induced in both the springs is approximately the same

$$\therefore \tau_1 = \tau_2$$

$$\text{i.e., } \frac{8F_1D_1k_1}{\pi d_1^3} = \frac{8F_2D_2k_2}{\pi d_2^3}; \text{ if } k_1 = k_2$$

$$\text{Then, } \frac{F_1D_1}{d_1^3} = \frac{F_2D_2}{d_2^3} \quad \text{---- (1)}$$

$$\text{Also } y_1 = y_2$$

$$\therefore \frac{8F_1D_1^3i_1}{d_1^4G} = \frac{8F_2D_2^3i_2}{d_2^4G}$$

$$\text{i.e., } \frac{F_1D_1^3i_1}{d_1^4} = \frac{F_2D_2^3i_2}{d_2^4}$$

Since solid lengths are equal

$$i_1d_1 = i_2d_2$$

$$\therefore \frac{F_1D_1^3(i_1d_1)}{d_1^4.d_1} = \frac{F_2D_2^3(i_2d_2)}{d_2^4 d_2}$$

$$\therefore \frac{F_1D_1^3}{d_1^5} = \frac{F_2D_2^3}{d_2^5} \quad \text{---- (2)}$$

Dividing (2) by (1)

$$\frac{D_1^2}{d_1^2} = \frac{D_2^2}{d_2^2}$$

$$\therefore \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = c \qquad \text{---- (3)}$$

∴ Spring index for both the springs is same

From equation (1) and (3) we have

$$\frac{F_1.c}{d_1^2} = \frac{F_2.c}{d_2^2}$$

$$\therefore \quad \frac{F_1}{F_2} = \frac{d_1^2}{d_2^2}$$

From Fig. 20.10 (DDHB Vol.I) we can find that the radial clearance between the two springs,

$$2C = (D_1 - D_2) - \left(\frac{2d_1}{2} + \frac{2d_2}{2}\right)$$

$$C = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Take the std radial clearance as $\frac{d_1 - d_2}{2}$

$$\frac{D_1}{2} - \frac{D_2}{2} - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

$$\therefore \quad \frac{D_1 - D_2}{2} = \frac{d_1}{2} - \frac{d_2}{2} + \frac{d_1}{2} + \frac{d_2}{2}$$

$$\text{i.e.,} \quad \frac{D_1 - D_2}{2} = d_1 \qquad \text{---- (4)}$$

$$\text{But } c = \frac{D_1}{d_1} = \frac{D_2}{d_2}$$

∴ $D_1 = cd_1$ and $D_2 = cd_2$, sub in equation (4)

$$\therefore \quad \frac{cd_1 - cd_2}{2} = d_1$$

$$cd_1 - cd_2 = 2d_1$$

$$\text{i.e.,} \quad cd_1 - 2d_1 = cd_2$$

$$d_1(c - 2) = cd_2$$

$$\therefore \quad \mathbf{\frac{d_1}{d_2} = \frac{c}{c-2}}$$ where c is spring index

Springs in Series

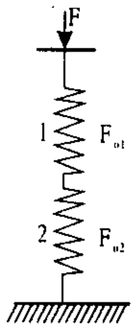


Fig. 3.9 a

$$F_{01} = \frac{F_1}{y_1}$$

$$F_{02} = \frac{F_2}{y_2}$$

$$F = F_1 = F_2$$

$$y = y_1 + y_2$$

$$\therefore \frac{F}{F_0} = \frac{F_1}{F_{01}} + \frac{F_2}{F_{02}}$$

$$\therefore \mathbf{\frac{1}{F_0} = \frac{1}{F_{01}} + \frac{1}{F_{02}}} \quad (\because \; F = F_1 = F_2)$$

where F_0 = combined stiffness

Springs in parallel

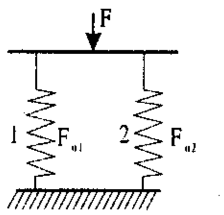


Fig. 3.9 b

$$F = F_1 + F_2$$

$$y = y_1 = y_2$$

$$\therefore F_0 y = F_{01}\, y_1 + F_{02}\, y_2$$

$$\therefore \mathbf{F_0 = F_{01} + F_{02}}$$

where F_0 = combined stiffness

CONCENTRIC SPRINGS

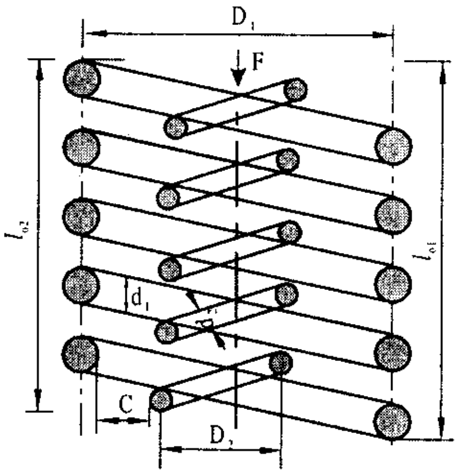


Fig. 3.10

D_1 = Mean diameter of coil of outer spring

D_2 = Means diameter of coil of inner spring

d_1 = Wire diameter of outer spring

d_2 = Wire diameter of inner spring

l_{01} = Free length of outer spring

l_{02} = Free length of inner spring

F = Total load on the springs

F_1 = Load on the outer spring

F_2 = Load on the inner spring

Type : 1

If both springs have same free length then

$$F_1 + F_2 = F \quad \text{---- (1)}$$

$$y_1 = y_2 \quad \text{---- (2)}$$

Equal free length

Example 3.19

One helical spring is rested inside another. The dimensions are as tabulated. Both springs have same free length and carry a total load of 2500 N. Take G = 80 GPa. Determine for each spring (i) Maximum load carried (ii) deflection and (iii) Maximum shear stress.

Particulars	*Outer spring*	*Inner spring*
Number of active coils	**6**	**10**
Wire diameter (mm)	**12.5**	**6.2**
Mean coil diameter (mm)	**87.5**	**56.3**

Solution :

$i_1 = 6$ $i_2 = 10$

$d_1 = 12.5\,mm$ $d_2 = 6.2\,mm$

$D_1 = 87.5\,mm$ $D_2 = 56.3\,mm$

$F = 2500\,N,$ $G = 80\,GPa = 80000\,MPa$

Equal free length

i) Maximum load on each spring

Solution :

Since both springs have same equal free lengths

$$F_1 + F_2 = 2500\text{ N} \quad \text{---- (1)}$$

$$\frac{F_1}{F_2} = \left(\frac{D_2}{D_1}\right)^3 \cdot \left(\frac{d_1}{d_2}\right)^4 \cdot \left(\frac{i_2}{i_1}\right)\left(\frac{G_1}{G_2}\right) \quad \text{---- 20.69}$$

$$= \left(\frac{56.3}{87.5}\right)^3 \cdot \left(\frac{12.5}{6.2}\right)^4 \cdot \left(\frac{10}{6}\right)\left(\frac{80000}{80000}\right)$$

$$\mathbf{F_1 = 7.3354\,F_2} \quad \text{---- (2)}$$

Sub in (1) $7.3354\,F_2 + F_2 = 2500$

$\therefore\ F_2 = $ **300 N and $F_1 = 2200$ N**

ii) Deflection

$y_1 = y_2$ ($\because$ Equal free length)

$$\therefore\ y_1 = \frac{8F_1D_1^3i_1}{d_1^4G_1} = \frac{8\times2200\times87.5^3\times6}{12.5^4\times80000}$$

$y_1 = 36.22\text{ mm} = y_2$

iii) Maximum shear stress induced

Shear stress on outer spring $\tau_1 = \dfrac{8F_1D_1k_1}{\pi d_1^3}$; $\qquad c_1 = \dfrac{D_1}{d_1} = \dfrac{87.5}{12.5} = 7$

$$\therefore\ k_1 = \frac{4c_1-1}{4c_1-4}+\frac{0.615}{c_1} = \frac{4\times7-1}{4\times7-4}+\frac{0.615}{7} = 1.2128$$

$$\therefore\ \tau_1 = \frac{8\times2200\times87.5\times1.2128}{\pi\times12.5^3} = 304.4\text{ N/mm}^2$$

Shear stress on inner spring $\tau_2 = \dfrac{8F_2D_2k_2}{\pi d_2^3}$ $\quad c_2 = \dfrac{D_2}{d_2} = \dfrac{56.3}{6.2} = 9.08$

$$\therefore\ k_2 = \frac{4\times9.08-1}{4\times9.08-4}+\frac{0.615}{9.08} = 1.1605$$

$$\therefore\ \tau_2 = \frac{8\times300\times56.3\times1.1605}{\pi\times6.2^3} = 209.4\text{ N/mm}^2$$

Unequal free length

Example 3.20

The table below gives the particulars of a concentric helical spring. If the spring is subjected to an axial load of 400 N, determine for each spring, (i) Change in length (ii) Amount of load carried (iii) Torsional shear stress induced. Take G = 84 GPa = 84000 MPa.

Particulars	***Inner spring***	***Outer spring***
Mean coil diameter (mm)	**30**	**40**
Diameter of wire (mm)	**4**	**4.875**
Number of active turns	**8**	**10**
Free length (mm)	**75**	**90**

Data :

$D_2 = 30\text{ mm};\ D_1 = 40\text{ mm};\ d_2 = 4\text{ mm};\ d_1 = 4.875\text{ mm}$

$i_2 = 8;\ i_1 = 10;\ l_{02} = 75\text{ mm};\ l_{01} = 90\text{ mm};\ G = 84000\text{ N/mm}^2;\ F = 400\text{ N}$

Solution :

Unequal free length

i) Amount of load carried

The load required to compress the outer spring by 15 mm is $y = \dfrac{8FD^3i}{d^4G}$

$$\text{i.e.,}\quad 15 = \frac{8\times F\times 40^3\times 10}{4.875^4\times 84000}$$

$$\mathbf{F = 139\ N}$$

$$\therefore\ \text{Remaining load } F_R = 400 - 139 = 261\ N$$

This load will be shared by the springs for further equal deflection.

$$\text{i.e.,}\quad F_R = F_{R1} + F_{R2}$$

$$\therefore\ F_{R1} + F_{R2} = 261 \qquad \text{---- (1)}$$

$$\frac{F_{R1}}{F_{R2}} = \left(\frac{D_2}{D_1}\right)^3\left(\frac{d_1}{d_2}\right)^4\cdot\left(\frac{i_2}{i_1}\right)\left(\frac{G_1}{G_2}\right) \qquad \text{---- 20.69}$$

$$= \left(\frac{30}{40}\right)^3\left(\frac{4.875}{4}\right)^4\left(\frac{8}{10}\right)\left(\frac{84000}{84000}\right)$$

$$\therefore\ \mathbf{F_{R1} = 0.7446\ F_{R2}} \qquad \text{---- (2)}$$

Sub in equation (1), $0.7446\ F_{R2} + F_{R2} = 261$

$$\therefore\ \mathbf{F_{R2} = 149.6\ N}$$

$$\mathbf{F_{R1} = 111.4\ N}$$

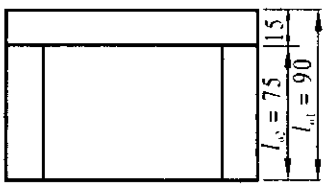


Fig. 3.11

$\therefore$ Total load on the outer spring $F_1 = F_{R1} + 139 = 111.4 + 139$

$$\therefore\ \mathbf{F_1 = 250.4\ N}$$

Load on the inner spring $\mathbf{F_2 = 149.6\ N}$

ii) Change in length

$$y_2 = \frac{8F_2D_2^3i_2}{d_2^4G} = \frac{8\times 149.6\times 30^3\times 8}{4^4\times 84000}$$

Change in length for inner spring $y_2 = \mathbf{12.02\ mm}$

Change in length for outer spring $y_1 = 15 + 12.02 = \mathbf{27.02\ mm}$

iii) Torsional shear stress induced

Shear stress on the outer spring $\tau_1 = \dfrac{8F_1D_1k_1}{\pi d_1^3}$; $\qquad c_1 = \dfrac{D_1}{d_1} = \dfrac{40}{4.875} = 8.2$

$$k_1 = \frac{4c_1 - 1}{4c_1 - 4} + \frac{0.615}{c_1} = \frac{4\times 8.2 - 1}{4\times 8.2 - 4} + \frac{0.618}{8.2} = 1.179$$

$$\tau_1 = \frac{8\times 250.4\times 40\times 1.179}{\pi\times 4.875^3} = 259.55\ N/mm^2$$

Shear stress on the inner spring $\tau_2 = \frac{8F_2D_2k_2}{\pi d_2^3}$; $\quad c_2 = \frac{D_2}{d_2} = \frac{30}{4} = 7.5$

$$k_2 = \frac{4\times7.5-1}{4\times7.5-4}+\frac{0.615}{7.5} = 1.197$$

$$\therefore \quad \tau_2 = \frac{8\times149.6\times30\times1.197}{\pi\times4^3} = 213.75 \text{ N/mm}^2$$

Note :

In concentric springs if both are same equal free length, same stress etc,

then **std. clearance** $\frac{d_1}{d_2} = \frac{c}{c-2}$

Example 3.21

Design a concentric spring for an air craft engine valve to exert a maximum force of 5000 N under a deflection of 40 mm. Both the springs have same free length, solid length and are subjected to equal maximum shear stress of 0.85 GPa spring index for both spring is 6. Assume G = 80 GPa and diametral clearance to be equal to difference between wire diameters.

Data :

$$F = 5000\text{ N};\; y_1 = y_2 = 4\text{ cm} = 40\text{ mm};\; l_{01} = l_{02}$$

$$\tau_1 = \tau_2 = 0.85\text{ GPa} = 850\text{ N/mm}^2;\; c_1 = c_2 = 6;\; G = 80\text{ GPa} = 80000\text{ N/mm}^2$$

Solution :

Since both springs are stressed to the same value,

$$\frac{F_1}{F_2} = \left(\frac{D_2}{D_1}\right)\left(\frac{d_1}{d_2}\right)^3\left(\frac{k_1}{k_2}\right) \qquad \text{---- 20.70}$$

$$= \left(\frac{c_2d_2}{c_1d_1}\right)\left(\frac{d_1}{d_2}\right)^3\left(\frac{k_1}{k_2}\right) \quad (\because\ k_1 = k_2;\ c_1 = c_2)$$

$$\therefore \quad \frac{F_1}{F_2} = \left(\frac{d_1}{d_2}\right)^2$$

Since std. clearance

$$\frac{d_1}{d_2} = \frac{c}{c-2} = \frac{6}{6-2} = 1.5$$

$$\therefore \quad \frac{F_1}{F_2} = (1.5)^2 = 2.25$$

$$\therefore \quad F_1 = 2.25F_2 \qquad \text{----(1)}$$

$$\text{Also } F_1 + F_2 = 5000 \qquad \text{----(2)}$$

Sub in (1) in equation (2)

$$2.25\,F_2 + F_2 = 5000$$

$$\therefore\ \mathbf{F_2 = 1538.5\ N\ and\ F_1 = 3461.5}$$

a) Design of Outer spring

i) Diameter of spring wire

$$\tau_1 = \frac{8F_1D_1k_1}{\pi d_1^3} \qquad k_1 = \frac{4c_1-1}{4c_1-4}+\frac{0.615}{c_1} = 1.2525$$

$$850 = \frac{8\times 3461.5\times 6d_1\times 1.2525}{\pi d_1^3} \qquad c_1 = \frac{D_1}{d_1}\ \therefore\ D_1 = 6\,d_1$$

$$\therefore\ d_1 = 8.82\ \text{mm}$$

Standard wire diameter $\mathbf{d_1} = \mathbf{9\ mm}$ From Table 20.12 (Old DDHB)

ii) Diameter of coil

Mean diameter of coil $D_1 = 6\,d_1 = 6\times 9 = 54$ mm

Outer diameter of coil $D_{01} = D_1 + d_1 = 54 + 9 = 63$ mm

Inner diameter of coil $D_{i1} = D_1 - d_1 = 54 - 9 = 45$ mm

iii) Number of coils or turns

$$\text{Deflection}\quad y_1 = \frac{8F_1D_1^3 i_1}{d_1^4 G_1}$$

$$\text{i.e.,}\quad 40 = \frac{8\times 3461.5\times 54^3\times i_1}{9^4\times 80000}$$

$$\therefore\ i_1 = 4.815$$

$\therefore$ Number of active turns in the outer spring $i_1 = 5$

iv) Free length

$$l_{01} = (i_1 + n)\,d_1 + y_1 + a$$

Assume squared and ground end

$\therefore$ Number of additional coil $n = 2$

Total number of turns $i_1' = i_1 + n = 5 + 2 = 7$

Clearance a = 25% of maximum deflection $= 0.25\times 40 = 10$ mm

$$\therefore\ l_{01} \geq 7\times 9 + 40 + 10$$

$$\geq 113\ \text{mm}$$

v) Pitch

$$p_1 = \frac{l_{01}-2d_1}{i_1} = \frac{113-2\times 9}{5} = 19\ \text{mm}$$

vi) Rate of spring

$$F_{01} = \frac{F_1}{y_1} = \frac{3461.5}{40} = 86.5375\ \text{N/mm}$$

vii) Total length of outer wire

$$l_1 = \pi D_1 i_1' = \pi \times 54 \times 7 = 1187.522 \text{ mm}$$

b) Design of inner spring

i) Diameter of spring wire

$$\tau_2 = \frac{8F_2 D_2 k_2}{\pi d_2^3} \qquad k_2 = \frac{4c_2 - 1}{4c_2 - 4} + \frac{0.615}{c_2}$$

$$\text{i.e.,}\quad 850 = \frac{8 \times 1538.5 \times 6d_2 \times 1.2525}{\pi d_2^3} \qquad = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

$$\therefore\ d_2 = 5.885 \text{ mm} \qquad c_2 = \frac{D_2}{d_2} \quad \therefore\ D_2 = 6d_2$$

Standard diameter of wire from Table 20.12 (Old DDHB)

$$d_2 = 6\text{mm}$$

ii) Diameter of coil

Mean diameter of coil $D_2 = 6d_2 = 6 \times 6 = 36$ mm

Outer diameter of coil $D_{02} = D_2 + d_2 = 36 + 6 = 42$ mm

Inner diameter of coil $D_{i2} = D_2 - d_2 = 36 - 6 = 30$ mm

iii) Number of coils or turns

$$\text{Deflection}\quad y_2 = \frac{8F_2 D_2^3 i_2}{d_2^4 G_2}$$

$$\text{i.e.,}\quad 40 = \frac{8 \times 1538.5 \times 36^3 \times i_2}{6^4 \times 80000}$$

$$\therefore\ i_2 = 7.22$$

$\therefore$ Number of active turns $i_2 = 8$

iv) Free length

$$lo_2 \geq (i_2 + n)\, d_2 + y_2 + a$$

Assume squared and ground end

$\therefore$ Number of additional coil $n = 2$

Total number of turns or coils $i'_2 = i_2 + 2 = 8 + 2 = 10$

Clearance a = 25% of maximum deflection $= 0.25\, y_2 = 0.25 \times 40 = 10$ mm

Take the clearance as 13 mm since equal free length

$$\therefore\ lo_2 \geq 10 \times 6 + 40 + 13$$

$$\geq 113 \text{ mm}$$

v) Pitch

$$p_2 = \frac{l_{o2} - 2d_2}{i_2} = \frac{113 - 2 \times 6}{8} = 12.625 \text{ mm}$$

vi) Rate of spring

$$F_{o2} = \frac{F_2}{y_2} = \frac{1538.5}{40} \doteq 38.4625 \text{ mm}$$

vii) Total length of inner spring wire

$$l_2 = \pi D_2 i'_2 = \pi \times 36 \times 10 = 1130.973 \text{ mm}$$

Spring Specifications

Particulars	*Outer spring*	*Inner spring*
Wire diameter	$d_1 = 9$ mm	$d_2 = 6$ mm
Mean diameter of coil	$D_1 = 54$ mm	$D_2 = 36$ mm
Free length	$l_{o1} = 113$ mm	$l_{o2} = 113$ mm
Total number of turns	$i'_1 = 7$	$i'_2 = 10$
Style of ends	Squared and ground	Squared and ground
Pitch	$p_1 = 19$ mm	$p_2 = 12.625$ mm
Rate of spring	$F_{o1} = 86.5375$ N/mm	$F_{o2} = 38.4625$ N/mm

Example 3.22

A composite spring has two closed coiled springs in series. Each spring has a mean diameter of 10 times its wire diameter. One spring has 20 active coils of wire diameter 3 mm. Find the diameter of other spring, if it has 15 active coils and the stiffness of composite spring is 1 N/mm. Also find the greatest axial load that can be applied and the corresponding extension for the maximum shear stress of 0.3 GPa and G = 80 GPa.

Data :

$$c_1 = c_2 = 10; \; d_1 = 3 \text{ mm}; \; i_1 = 20; \; i_2 = 15; \; F_o = 1 \text{ N/mm};$$
$$\tau = 0.3 \text{ GPa} = 300 \text{ N/mm}^2; \; G = 80 \text{ GPa} = 80 \times 10^3 \text{ N/mm}^2$$

Solution :

Let the springs in series be as shown in figure 2.12.

Since the springs are in series

$$\frac{1}{F_o} = \frac{1}{F_{o1}} + \frac{1}{F_{o2}}$$

$$c_1 = \frac{D_1}{d_1} \quad \therefore \; D_1 = c_1 d_1 = 10 \times 3 = 30 \text{ mm}$$

$$c_2 = \frac{D_2}{d_2} \quad \therefore \; D_2 = c_2 d_2 = 10 \, d_2$$

$$\frac{1}{1} = \frac{8 i_1 D_1^3}{d_1^4 G_1} + \frac{8 i_2 D_2^3}{d_2^4 G_2} = \frac{8 \times 20 \times 30^3}{3^4 \times 80 \times 10^3} + \frac{8 \times 15 \times (10 d_2)^3}{d_2^4 \times 80 \times 10^3}$$

$\therefore \; d_2 = 4.5$ mm = Diameter of second spring wire

$$\text{Now, } k_1 = \frac{4c_1-1}{4c_1-4}+\frac{0.615}{c_1} = \frac{4\times 10-1}{4\times 10-4}+\frac{0.615}{10} = 1.1448 = k_2$$

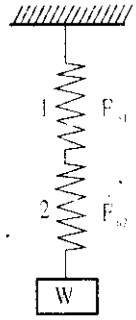


Fig. 3.12

Shear stress in the first spring wire $\tau_1 = \dfrac{8F_1D_1k_1}{\pi d_1^3}$

$$\text{i.e., } 300 = \frac{8\times F_1\times 30\times 1.1448}{\pi\times 3^3}$$

$$\therefore\ F_1 = 92.62 \text{ N}$$

Shear stress in the second spring wire $\tau_2 = \dfrac{8F_2D_2k_2}{\pi d_2^3}$

$$\text{i.e., } 300 = \frac{8F_2\times 45\times 1.1448}{\pi\times 4.5^3} \quad (\because D_2 = 10\,d_2 = 10\times 4.5 = 45 \text{ mm})$$

$$\therefore\ F_2 = 208.4 \text{ N}$$

Permissible load is smaller among the above two values.

$\therefore$ Permissible load **F = 92.62 N**

$$\text{Also rate of spring } F_o = \frac{F}{y}$$

$$\text{i.e., } 1 = \frac{92.62}{y}$$

$\therefore$ y = 92.62 mm = Extension of the springs.

3.14 HELICAL SPRINGS OF NON-CIRCULAR CROSS-SECTIONS

The use of rectangular or square cross-section spring wire shown in Fig. 3.13 a and b is not recommended unless space limitations make it necessary. Springs of non-circular cross section are not as strong as the springs made of circular cross section wire. The wire becomes trapezoidal when the coil is formed. When space is severely limited, the use of concentric springs should be considered.

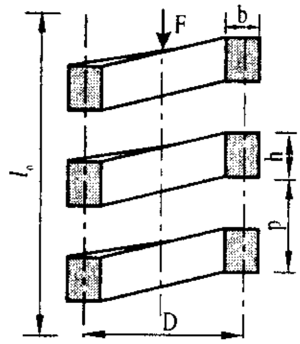
Let b = Width of rectangular c/s spring [side of rectangle perpendicular to the axis]

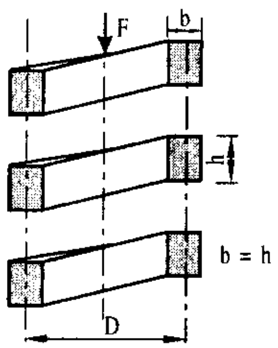
h = Axial height of rectangular c/s spring [side of rectangle parallel to the axis]

D = Mean diameter of coil; c = spring index

The equation for the stress produced and deflection in a rectangular cross section wire are given below.

$$\text{Shear stress } \tau' = \frac{kFD(1.5h+0.9b)}{b^2h^2} \quad \text{---- 20.34 a}$$





(a) [DDHB 20.7a]: Rectangular cross-section spring *Fig. 3.13* *(b) Square cross- section spring*

$$k = \text{Stress factor} = \frac{4c-1}{4c-4} + \frac{0.615}{c} \quad \text{---- } 20.36$$

$$c = \text{Spring index} = \frac{D}{b} \quad \text{if } b < h$$

$$= \frac{D}{h} \quad \text{if } h < b \quad \text{---- } 20.38$$

$$\text{Deflection } y = \frac{2.83i\,FD^3\left(b^2+h^2\right)}{b^3h^3G} \quad \text{----- } 20.39$$

For square cross-section spring, $b = h$

$$\text{Shear stress } \tau' = \frac{2.4kFD}{h^3} \quad \text{---- } 20.43$$

$$\text{Deflection } y = \frac{5.66i\,FD^3}{h^4G} \quad \text{---- } 20.44$$

Example 3.23

A rectangular section helical spring is mounted to a buffer to sustain a maximum load of 30 kN. The deflection under load is limited to 100 mm. The spring is made of chrome-vanadium steel with a reliability of 1.5. The longer side of the rectangle is 2 times the shorter side and the spring is wound with longer side parallel to the axis. The spring index is 10. Design the spring and draw a conventional sketch.

Data :

$$F = 30\text{ kN} = 30000\text{ N}; y = 100\text{ mm}; \text{ Fos} = 1.5; \; h = 2b; \; c = 10$$

Solution :

From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB) for Chrome vanadium steel, $\tau_y = 0.69\text{ GPa} = 690\text{ N/mm}^2$

$$G = 79.34\text{ GPa} = 79.34 \times 10^3\text{ N/mm}^2$$

$$\tau = \frac{\tau_y}{FOS} = \frac{690}{1.5} = 460\ N/mm^2 = \tau'$$

i) Cross-section of spring

$$\text{Shear stress } \tau' = \frac{kFD(1.5h + 0.9b)}{b^2h^2} \quad \text{---- 20.34 a}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times10-1}{4\times10-4} + \frac{0.615}{10} = 1.1448$$

$$\text{Spring index } c = \frac{D}{b} \quad \text{since } b < h$$

$$\therefore D = cb = 10\,b$$

$$\therefore 460 = \frac{1.1448\times30000\times10b[1.5\times2b + 0.9b]}{b^2(2b)^2}$$

$\therefore$ Width of spring $b = 26.98\ mm = 27\ mm$

Height of spring $h = 2b = 2\times27 = 54\ mm$

ii) Diameter of coil

Mean coil diameter $D = 10b = 10\times27 = 270\ mm$

Outer coil diameter $D_o = D + b = 270 + 27 = 297\ mm$

Inner coil diameter $D_i = D - b = 270 - 27 = 243\ mm$

iii) Number of coils or turns

$$\text{Deflection } y = \frac{2.83i\,FD^3(b^2+h^2)}{b^3h^3G} \quad \text{---- 20.39}$$

$$\text{i.e., } 100 = \frac{2.83i\times30000\times270^3[27^2+54^2]}{(27^3)(54^3)(79.34\times1000)}$$

$$i = 4.037 \approx 5$$

$\therefore$ Number of active turns $i = 5$

iv) Free length

$$l_o \geq (i+n)h + y + a$$

Assume squared and ground end

$\therefore$ Number of additional coils $n = 2$

Total number of turns $i' = i + n = 5 + 2 = 7$

Clearance $a = 25\%$ of deflection $= 0.25 \times 100 = 25$ mm

$$\therefore l_o \geq 7\times54 + 100 + 25$$

$$\geq 503\ mm$$

v) Pitch

$$p = \frac{l_o - 2h}{i} = \frac{503 - 2\times54}{5} = 79\ mm$$

vi) Rate of spring

$$F_o = \frac{F}{y} = \frac{30000}{100} = 300 \text{ N/mm}$$

vii) Total length of wire

$$l = \pi \text{ Di}' = \pi \times 270 \times 7 = 5937.6 \text{ mm}$$

viii) Sketch similar to figure 2.13 a

Spring specifications

i) Material - chrome - vanadium steel
ii) Wire cross-section - width b = 27 mm, height h = 54 mm
iii) Mean coil diameter D = 270 mm
iv) Free length l_o = 503 mm
v) Total number of turns i' = 7
vi) Style of ends – squared and ground
vii) Pitch p = 79 mm
viii) Rate of spring F_o = 300 N/mm

Example 3.24

A diesel engine weighs 800 kN is mounted on 16 springs in order to protect the building from vibration. The section of the spring wire is rectangular with side ratio 1.8. One spring has four effective coils. Spring index 6. Determine (i) Section of spring so that longer side is parallel to the axis. (ii) Deflection under load when the engine is stationary (iii) Maximum coil diameter and (iv) Shear stress induced if shorter side is parallel to the axis. Take τ = 0.3 GPa and G = 80 GPa

Data :

$$W = 800 \text{ kN} = 8 \times 10^5 \text{ N}; \quad \text{Number of springs} = 16$$

$$i = 4; \quad \frac{h}{b} = 1.8; \quad c = 6; \quad \tau = 0.3 \text{ GPa} = 300 \text{ N/mm}^2$$

$$G = 80 \text{ GPa} = 80 \times 10^3 \text{ N/mm}^2$$

Solution :

$$\text{Axial load on each spring } F = \frac{\text{Weight of engine}}{\text{Number of springs}} = \frac{8 \times 10^5}{16} = 50000 \text{ N}$$

i) Cross-section of spring

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525 \qquad \text{---- 20.36}$$

$$c = \frac{D}{b} \quad \text{since } b < h$$

$$\therefore D = cb = 6\,b$$

$$\text{Shear stress } \tau' = \frac{kFD[1.5h + 0.9b]}{b^2h^2} \qquad \text{---- 20.34 a}$$

$$\text{i.e., } 300 = \frac{1.2525 \times 50000 \times 6b[1.5 \times 1.8b + 0.9b]}{(b^2)(1.8b)^2}$$

$\therefore$ Width of spring $b = 37.3 \approx 37.5$ mm

Height of spring $h = 1.8b = 1.8 \times 37.5 = 67.5$ mm

ii) Maximum coil diameter

Mean coil diameter $D = cb = 6 \times 37.5 = 225$ mm

Outer diameter of coil $D_o = D + b = 225 + 37.5 = 262.5$ mm = Maximum coil diameter

iii) Deflection under the load

$$y = \frac{2.83 i F D^3 (b^2 + h^2)}{b^3 h^3 G} \quad \text{---- 20.39}$$

$$= \frac{2.83 \times 4 \times 50000 \times 225^3 (37.5^2 + 67.5^2)}{(37.5^3)(67.5^3)(80 \times 10^3)} = 29.6276 \text{ mm}$$

iv) Shear stress induced if shorter side is parallel

$\therefore h = 37.5$ mm and $b = 67.5$ mm

$$\text{Shear stress } \tau' = \frac{1.2525 \times 50000 \times 225[1.5 \times 37.5 + 0.9 \times 67.5]}{67.5^2 \times 37.5^2} = 257.3 \text{ N/mm}^2$$

Example 3.25

A railway car weighing 18 kN and moving at a speed of 72 m/min is brought to rest by a buffer consisting of 2 helical compression springs of *square cross-section*. In bringing the car to rest the spring under goes a deflection of 0.25 m. The allowable shear stress for the steel wire is 0.3 GPa. Spring index = 6. Design the spring and draw a conventional sketch. Take G = 84 GPa.

Data :

$W = 18$ kN $= 18 \times 10^3$ N; $v = 72$ m/min $= \frac{72 \times 1000}{60} = 1200$ mm/sec

Number of springs = 2; $y = 0.25$ m $= 250$ mm; $c = 6$

$\tau = 0.3$ GPa $= 300$ N/mm^2; $G = 84$ GPa $= 84 \times 10^3$ N/mm^2

Solution :

$$\text{Kinetic energy (KE)} = \frac{1}{2} M v^2 = \frac{1}{2} \frac{W}{g} v^2 = \frac{1}{2} \times \frac{18 \times 10^3}{9810} \times (1200)^2 = 1321100.917 \text{ Nmm}$$

$$\therefore \text{ Energy stored by each spring} = \frac{\text{Total kinetic energy}}{\text{Number of springs}} = \frac{1321100.917}{2} = 660550.46 \text{ Nmm}$$

Also, energy stored by each spring $U = \frac{1}{2} Fy$

$$\text{i.e., } 660550.46 = \frac{1}{2} F \times 250$$

$\therefore$ Axial force an each spring $F = 5284.4\,N$

i) Cross section of spring

$$\text{Shear stress } \tau' = \frac{2.4\,kFD}{h^3} \qquad \text{---- 20.43}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525$$

$$\text{Spring index } c = \frac{D}{h} \quad \therefore \ D = ch = 6\,h$$

$$\text{i.e., } 300 = \frac{2.4\times1.2525\times5284.4\times6h}{h^3}$$

$h = 17.824$ mm $\cong$ 18 mm = b ($\because$ square c/s)

$\therefore$ Width of spring b = 18 mm

Height of spring h = 18 mm

ii) Diameter of coil

Mean diameter of coil $D = 6\,h = 6 \times 18 = 108$ mm

Outer diameter of coil $D_o = D + h = 108 + 18 = 126$ mm

Inner diameter of coil $D_i = D - h = 108 - 18 = 90$ mm

iii) Number of coils or turns

$$\text{Deflection } y = \frac{5.66i\,FD^3}{h^4G} \qquad \text{---- 20.44}$$

$$\text{i.e., } 250 = \frac{5.66i\times5284.4\times108^3}{18^4\times84\times10^3}$$

$\therefore$ $i = 58.5$

i.e., Number of active turns $i = 59$

iv) Free length

$$l_o \geq (i+n)\,h + y + a$$

Assume squared and ground end

$\therefore$ Number of additional coil $n = 2$

Total number of coils $i' = i + n = 59 + 2 = 61$

Clearance a = 25% of maximum deflection = $0.25\,y = 0.25 \times 250 = 62.5$ mm

$\therefore$ $l_o \geq 61 \times 18 + 250 + 62.5$

≥ 1410.5 mm

v) Pitch

$$p = \frac{l_o - 2h}{i} = \frac{1410.5 - 2\times18}{59} = 23.3\text{ mm}$$

vi) Rate of spring

$$F_o = \frac{F}{y} = \frac{5284.4}{250} = 21.1376 \text{ N/mm}$$

vii) Total length of wire

$$l = \pi Di' = \pi \times 108 \times 61 = 20696.8 \text{ mm}$$

Spring specifications

i) Wire cross-section-width b = 18 mm, height h = 18 mm

ii) Mean coil diameter D = 108 mm

iii) Free length l_o = 1410.5 mm

iv) Total number of turns i' = 61

v) Style of ends – squared and ground

vi) Pitch p = 23.3 mm

vii) Rate of spring F_o = 21.1376 N/mm

3.15 LEAF SPRING

Leaf springs are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

Consider a single plate fixed at one end and loaded at the other end as shown in Fig. 3.14 a. This plate may be used as a flat spring.

h = thickness of plate

b = width of plate

l = length of plate

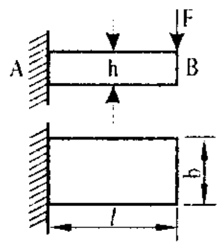


Fig. 3.14 a

We have $\frac{M}{I} = \frac{\sigma}{y}$

$$\therefore \quad \sigma = \frac{M}{I}.y = \frac{(Fl)\left(\frac{h}{2}\right)}{\left(\frac{bh^3}{12}\right)}$$

$$\therefore \quad \sigma = \frac{6Fl}{bh^2} \qquad \text{---- (i)}$$

The maximum deflection for a cantilever with point load at the free end $y = \frac{Fl^3}{3EI}$

$$\text{i.e.,} \quad y = \frac{Fl^3}{3E\frac{bh^3}{12}} = \frac{4Fl^3}{Ebh^3} = \frac{2\sigma l^2}{3Eh} \qquad \text{---- (ii)}$$

If the spring is not of cantilever type but like a simply supported beam as shown in Fig. 3.14b, with length 2 l and load 2 F, then

$$\sigma = \frac{M}{I}.y = \frac{(Fl)\left(\frac{h}{2}\right)}{\frac{bh^3}{12}} = \frac{6Fl}{bh^2}$$

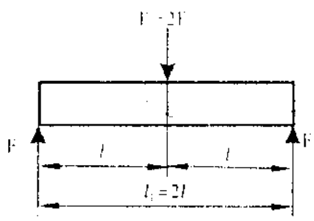


Fig. 3.14 b

Maximum deflection at the centre $y = \frac{F_1 l_1^3}{48EI} = \frac{(2F)(2l)^3}{48E\left(\frac{bh^3}{12}\right)} = \frac{4Fl^3}{Ebh^3}$

From the above equation it is concluded that a spring such as automobile spring [semi-elliptical spring] with length 2l and loaded in the centre by a load 2 F may be considered as a double cantilever.

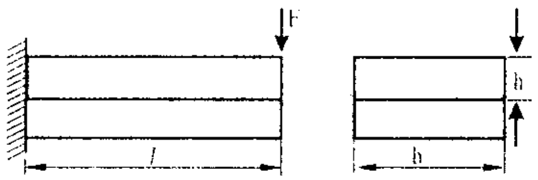


Fig. 3.14 c

If a plate of cantilever is cut into 'i' number of strips of width 'b' and these are placed as shown in Fig. 3.14 c, then the equations (i) and (ii) can be written as

$$\sigma = \frac{6Fl}{ibh^2} \qquad \text{---- (iii)}$$

$$y = \frac{4Fl^3}{Eibh^3} = \frac{2\sigma l^2}{3Eh} \qquad \text{---- (iv)}$$

Equations (iii) and (iv) gives the stress and deflection of a leaf of uniform cross-section. The stress for this spring is maximum at the support.

If a triangular plate is used as shown in Fig. 3.14 d, the stress will be uniform through out. If this triangular plate is cut into strips of uniform width and placed one below the other as shown in Fig. 3.14 e to form a graduated or laminated leaf spring

$$\text{then, } \sigma = \frac{6Fl}{ibh^2} \qquad \text{---- (v)}$$

$$y = \frac{6Fl^3}{Eibh^3} = \frac{\sigma l^2}{Eh} \qquad \text{---- (vi)}$$

From equations (iv) and (vi) that for the same deflection, the stress in the uniform cross-section leaves (i.e., full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own element curve. If the suffixes 'f' and 'g' are used to indicate the full length and graduated leaves, then

$$\sigma_f = \frac{3}{2}\,\sigma_g$$

i.e., $$\frac{6F_f l}{i_f bh^2} = \frac{3}{2}\left(\frac{6F_g l}{i_g bh^2}\right)$$

$$\therefore\ \frac{F_f}{F_g} = \frac{3}{2}\frac{i_f}{i_g} \qquad \text{---- (A)}$$

Adding '1' on both sides

$$\frac{F_f}{F_g} + 1 = \frac{3}{2}\frac{i_f}{i_g} + 1$$

i.e., $$\frac{F_f + F_g}{F_g} = \frac{(3i_f + 2i_g)}{2i_g}$$

$$\therefore\ F_g = (F_f + F_g)\left(\frac{2i_g}{3i_f + 2i_g}\right)$$

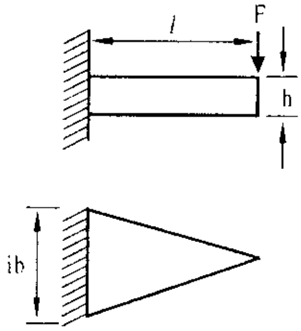


Fig. 3.14 d

i.e., $$F_g = F\left(\frac{2i_g}{3i_f + 2i_g}\right) \qquad [\because\ F_f + F_g = F = \text{Total load}]$$

From (A) $$\frac{F_g}{F_f} = \frac{2}{3}\left(\frac{i_g}{i_f}\right)$$

Adding '1' on both sides

$$\frac{F_g}{F_f} + 1 = \frac{2}{3}\left(\frac{i_g}{i_f}\right) + 1$$

i.e., $$\left(\frac{F_g + F_f}{F_f}\right) = \frac{2i_g + 3i_f}{3i_f}$$

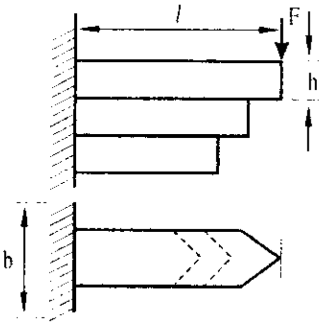


Fig. 3.14 e

$$\therefore\ F_f = (F_g + F_f)\left(\frac{3i_f}{2i_g + 3i_f}\right) = F\left(\frac{3i_f}{2i_g + 3i_f}\right)$$

Stress on full length leaves $\sigma_f = \dfrac{6F_f l}{i_f bh^2} = \dfrac{6l}{i_f bh^2}\cdot F\left(\dfrac{3i_f}{2i_g + 3i_f}\right)$

$$\therefore\ \sigma_f = \frac{18Fl}{(2i_g + 3i_f)bh^2}$$

Stress on graduated leaves $\sigma_g = \dfrac{2}{3}\sigma_f = \dfrac{2}{3}\cdot\dfrac{18Fl}{(2i_g + 3i_f)bh^2} = \dfrac{12Fl}{(2i_g + 3i_f)bh^2}$

Deflection in full length and graduated leaves is given by, $y = \frac{2\sigma_f l^2}{3Eh} = \frac{2l^2}{3Eh} \cdot \frac{18Fl}{(2i_g + 3i_f)bh^2}$

$$\therefore\ y = \frac{12Fl^3}{Ebh^3(2i_g + 3i_f)}$$

3.16 LEAF SPRING CONSTRUCTION

The most commonly used leaf spring is the semi-elliptical leaf spring. The semi-elliptical spring may be considered as two cantilevers and elliptical spring as four cantilevers. The stress induced in a semi-elliptical leaf spring is same as that of full elliptical leaf spring. But the deflection in a semi-elliptical leaf spring is equal to one half of full elliptical leaf spring. The unloaded spring is cambered. the magnitude of the camber being such that the spring is approximately straight under the full static load. The long leaf fastened to the supports is called the main leaf or master leaf. Its ends are bent to form an eye. If heavy loading is there, one or more full length leaves are provided below the master leaf. The other leaves of the spring are known as graduated leaves. The bundle of leaves are held together by means of a bolt passing through the centre or by a band shrunk around them at the centre. Since the band exerts a stiffening and strengthening effect, this should be allowed for by substracting the width of band from the overall length of the spring to obtain effective length for bending. In the case of bolt, the diameter of the bolt must be substracted from the width of the leaf when making the calculations for strength.

The U-bolts are used to hold the spring to the machine structure. The U-bolts create a stress concentration at the edge of the spring seat. A soft pad placed between the leaf and the seat will reduce the stress concentration. Rebound clips are located at intermediate positions in the length of spring. It helps to distribute to the graduated leaves some of the load of the rebound which otherwise would be taken by the master leaf alone. Ends of the spring are fastened to the supports by means of hinges. The spring is required to support the static and longitudinal loads and in addition driving and braking cause additional stresses.

Let $2F$ = Central load

L = Total length of springs between supports

l_b = Width of central band

l = Effective length = $\frac{L - l_b}{2}$

c = camber

y = deflection

i_f = Number of full length leaves = i'

i_g = Number of graduated leaves

i = Total number of leaves = $i_f + i_g$

σ_f = Maximum stress in the full-length leaves

σ = Maximum equalized stress

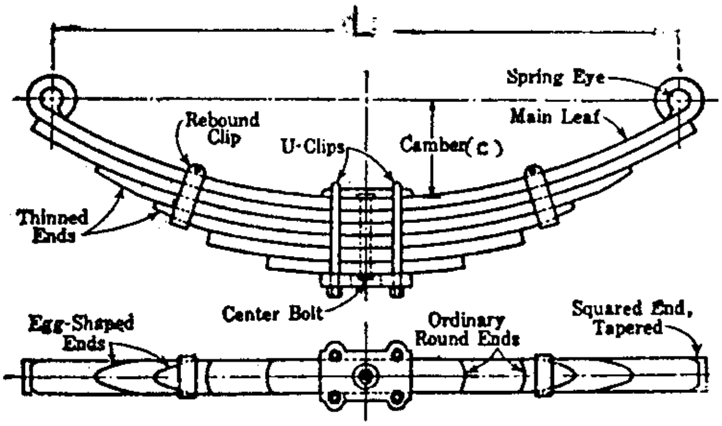


Fig. 3.15 [DDHB Fig. 20.1] : Laminated springs for automobiles

F_b = Load on the clip bolt or central band

E = Modulus of elasticity

b' = Width of each strip in a laminated spring

h = Height or thickness of laminated spring

α, β, α' and β' are constants from Table 20.3 (Old DDHB) or Table 20.7 (New DDHB)

3.17 DESIGN OF LEAF SPRINGS

In the design of leaf springs the following points are important.

i) The leaves are usually curved and the ends of the leaves are rounded or chamfered.

ii) The central bolt weakness the spring and hence shrunk bands are used for heavy springs.

iii) U-bolts which hold the spring to the seat, however, reduce the bending stresses at the plane of central bolt.

iv) The rebound-clips prevent the entry of dirt and grit during the rebound. Also it protects the spring from excessive stress.

v) Shackles at one or both ends permit small fluctuations in spring length

vi) If the spring is subjected to only static load or occasional load variations, a factor of safely of 1.5 is sufficient.

vii) For springs subjected to frequent load variations i.e., fatigue loading, a factor of safety of 1.5 based on endurance strength or 2 to 2.5 based on elastic limit is to be used.

Steps in design procedure

i) Size of spring leaf

Maximum stress in the full length leaves $\sigma_f = \dfrac{1.5\alpha Fl}{ib'h^2}$ ---- 20.14

Maximum equalised stress $\sigma = \frac{\alpha' Fl}{ib'h^2}$ ---- 20.18

Use σ equation if the leaves are pre-stressed or equally stressed otherwise use σ_f equation and find $b'h^2$

Maximum deflection $y = \frac{\beta Fl^3}{Eib'h^3}$ ---- 20.15

Using the above equation find $b'h^3$.

Now $\frac{b'h^3}{b'h^2}$ = h = Thickness of spring leaves

Substitute the value of h in $b'h^2$ and $b'h^3$ to find b'. Select the bigger value as the permissible value.

ii) Initial space (or) camber

Camber $c = \frac{\beta' Fl^3}{ib'Eh^3}$ ---- 20.16

iii) Load on the central band

Load on the clip bolt $F_b = \frac{2Fi_g i_f}{i(2i_g + 3i_f)}$ ---- 20.17

3.18 EQUALISED STRESS IN SPRING LEAVES (NIPPING)

The stress in the full length leaves is 50% greater than the stress in the graduated leaves. In order to utilise the material to the best advantage all the leaves should be equally stressed. This condition may be obtained in the following two ways.

1. By making the full length leaves of smaller thickness than the graduated leaves. In this way, the full length leaves will induce smaller bending stress due to small distance from the neutral axis to the edge of the leaf.
2. By giving a greater radius of curvature to the full length leaves than graduated leaves before the leaves are assembled to form a spring.

By doing so, a gap or clearance will be left between the leaves. This initial gap as shown by 'c' is called nip.

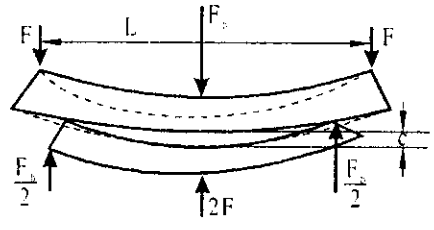


Fig. 3.16

When the central bolt, holding the various leaves together is tightened, the full length leaf will bend back as shown by the dotted lines in the Fig. 3.16 and have an initial stress in a direction opposite to that of normal load. The graduated leaves will have the initial stress in the same direction as that of normal load. When the load is gradually applied to the spring, the full length leaf is to first relieved of this initial stress and then stressed in the opposite direction. Therefore full length leaf will be stressed less than the graduated leaf. The initial gap between the leaves may be adjusted so that under maximum load condition, the stresses in all the leaves are equal.

To find the value of c, consider that under maximum load condition the stresses in all the leaves are equal. At maximum load, the total deflection of the graduated leaves will exceed the deflection of the full length leaves by an amount equal to the initial gap 'c'

$$\therefore \quad c = y_g - y_f$$

$$= \frac{6F_g l^3}{Ei_g bh^3} - \frac{4F_f l^3}{Ei_f bh^3} \qquad \text{---- (A)}$$

Since the stresses are equal

$$\sigma_g = \sigma_f$$

$$\text{i.e.,} \quad \frac{6F_g l}{i_g bh^2} = \frac{6F_f l}{i_f bh^2}$$

$$\therefore \quad \frac{F_g}{F_f} = \frac{i_g}{i_f}$$

$$\therefore \quad F_g = F_f . \frac{i_g}{i_f} = \frac{i_g}{i} . F$$

$$\text{and} \quad F_f = F_g . \frac{i_f}{i_g} = \frac{i_f}{i} F$$

Substituting in equation (A)

$$c = \frac{6Fl^3}{Eibh^3} - \frac{4Fl^3}{Eibh^3} = \mathbf{\frac{2Fl^3}{Eibh^3}}$$

The load on the clip bolts F_b required to close the gap is determined by the fact that the gap is equal to the initial deflection of full length and graduated leaves

$$\text{i.e.,} \quad c = y_f + y_g$$

$$\therefore \quad \frac{2Fl^3}{iEbh^3} = \frac{4l^3}{i_f Ebh^3} \times \frac{F_b}{2} + \frac{6l^3}{i_g Ebh^3} \times \frac{F_b}{2}$$

$$\frac{F}{i} = \frac{F_b}{i_f} + \frac{3}{2}\frac{F_b}{i_g}$$

$$= \frac{2F_b i_g + 3F_b i_f}{2i_f . i_g}$$

$$= \frac{F_b\left[2i_g + 3i_f\right]}{2i_f . i_g}$$

$$\therefore \quad F_b = \frac{2Fi_f i_g}{i\left[2i_g + 3i_f\right]}$$

Final stress σ = stress in the full length leaves – initial stress

$$= \frac{6F_f l}{i_f bh^2} - \frac{6l}{i_f bh^2} \times \frac{F_b}{2} = \frac{6Fl}{ibh^2}$$

Example 3.26

Determine the width and thickness of a flat spring carrying a central load of 5000 N. The deflection is limited to 100 mm. The spring is supported at both ends at a distance of 800 mm. The allowable stress is 300 N/mm² and modulus of elasticity 221 GPa. The spring is of constant thickness and varying width.

Data :

$F = 5000$ N; $y = 100$ mm; $2l = 800$ mm $\therefore$ $l = 400$ mm

$\sigma = 300$ Nmm²; $E = 221$ GPa $= 221 \times 10^3$ N/mm²

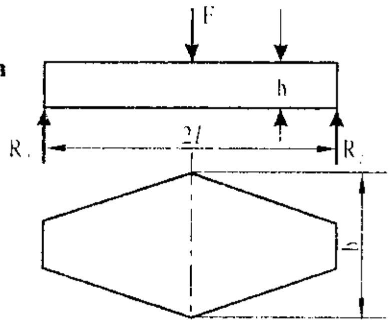


Fig. 3.17

Solution :

Since the spring is of constant thickness and varying width, it is as shown in Fig. 3.17 and from Table 20.1, $c_1 = 3$; $c_2 = 3$

Maximum stress in the spring $\sigma = \dfrac{c_1 Fl}{bh^2}$ ---- 20.1

$$\text{i.e.,} \quad 300 = \frac{3 \times 5000 \times 400}{bh^2}$$

$$\therefore \quad bh^2 = 20000 \qquad \text{---- (i)}$$

$$\text{Maximum deflection} \quad y = \frac{c_2 Fl^3}{Ebh^3} \qquad \text{---- 20.2}$$

$$\text{i.e.,} \quad 100 = \frac{3 \times 5000 \times 400^3}{221 \times 10^3 \times bh^3}$$

$$\therefore \quad bh^3 = 43438.914 \qquad \text{---- (ii)}$$

(ii) divided by (i) gives

$$\frac{bh^3}{bh^2} = h = \frac{43438.914}{20000} = 2.172$$

Take thickness of spring $h = 2.5$ mm

Width of spring at the centre,

$$\text{From equation (i)} \quad b = \frac{20000}{2.5^2} = 3200 \text{ mm}$$

From equation (ii) $b = \frac{43438.914}{2.5^3} = 2780\,mm$

Select the bigger value as the permissible Value $\therefore$ b = 3200 mm.

Example 3.27

Figure 3.18 shows a cantilever spring having four graduated leaves of each 5 mm thick and 50 mm wide. Determine the force required to cause a deflection of 25 mm at the end. What is the bending stress in the beam. Take E = 220 GPa

Data :

$i = 4$; $h = 5$ mm; $b' = 50$ mm; $y = 25$ mm $E = 220$ GPa $= 220 \times 10^3$ N/mm^2

Solution :

From Table 20.1 for constant width and varying depth $c_1 = 6$ and $c_2 = 8$

Maximum deflection $y = \frac{c_2 F l^3}{E i b' h^3}$ ---- 20.5

i.e., $25 = \frac{8 \times F \times 1200^3}{220 \times 10^3 \times 4 \times 50 \times 5^3}$

$\therefore$ F = 9.95 N = Applied load

Now load on the spring $F = \frac{\sigma i b' h^2}{c_1 l}$ ---- 20.4

i.e., $9.95 = \frac{\sigma \times 4 \times 50 \times 5^2}{6 \times 1200}$

$\therefore$ σ = 14.328 N/mm^2 = Bending stress in the beam

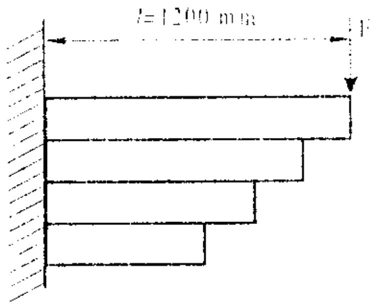


Fig. 3.18

Example 3.28

Determine the width and thickness of 6 leaves cantilever spring 300 mm long to carry a load of 1550 N with a deflection of 30 mm. The maximum stress in the spring should not exceed 0.330 GPa. Take E = 204 GPa

Data :

$i = 6$; $l = 300$ mm; $F = 1550$ N; $y = 30$ mm; $\sigma = 0.33$ GPa $= 330$ N/mm^2
$E = 204$ GPa $= 204 \times 10^3$ N/mm^2

Solution :

Since not mentioned anything about springs, assume uniform width and thickness.

$\therefore$ From Table 20.1 $c_1 = 6$ and $c_2 = 4$

Load on the spring $F = \frac{\sigma i b' h^2}{c_1 l}$ ---- 20.4

i.e., $1550 = \frac{330 \times 6 \times b' h^2}{6 \times 300}$

$\therefore$ $b'h^2 = 1409.09$ ---- (i)

$$\text{Maximum deflection } y = \frac{c_2 F l^3}{E i b' h^3} \quad \text{---- 20.5}$$

$$\text{i.e., } 30 = \frac{4 \times 1550 \times 300^3}{204 \times 10^3 \times 6 \times b' h^3}$$

$$b'h^3 = 4558.82 \quad \text{---- (ii)}$$

Equation (ii) divided by (i) gives

$$\frac{b' h^3}{b' h^2} = h = \frac{4558.82}{1409.09} = 3.235$$

take, the thickness of spring $h = 3.5$ mm

Substituting in equation (i) $b' = 115$ mm and

Substituting in equation (ii) $b' = 106.328$ mm

Select the bigger value as the permissible value ∴ Width of spring $b' = 115$ mm

---

Example 3.29

A locomotive spring has an over all length of 1100 mm and sustain a load of 75 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm. *All the leaves are to be stressed at 0.4 GPa* when fully loaded. The ratio of total spring depth to width is 2. Determine (i) Width and thickness of leaves. (ii) Initial space that must be provided between full length and graduated leaves before the band is applied. (iii) What load is exerted on the band after the spring is assembled.

Data :

$2F = 75 \text{ kN} = 75000 \text{ N} \;\; \therefore \;\; F = 37500 \text{ N}; \;\; i_f = 3 = i'; \;\; i_g = 15; \;\; l_b = 100 \text{ mm}$

$\sigma = 0.4 \text{ GPa} = 400 \text{ N/mm}^2 \;\; (\because \text{ Equally stressed}); \;\; \frac{ih}{b'} = 2; \;\; L = 1100 \text{ mm}$

Solution :

Assume $E = 206.8 \text{ GPa} = 206.8 \times 10^3 \text{ N/mm}^2$

$$\text{Effective length } l = \frac{L - l_b}{2} = \frac{1100 - 100}{2} = 500 \text{ mm}$$

Total number of leaves $i = i_f + i_g = 3 + 15 = 18$

From Table : 20.3 (Old DDHB) or Table 20.7 (New DDHB)

$$\alpha' = 6 : \beta' = 2$$

$$\alpha = \beta = \frac{12}{2 + r} = \frac{12}{2 + \frac{i'}{i}} = \frac{12}{2 + \frac{3}{18}} = 5.538$$

$$\frac{ih}{b'} = 2; \text{ i.e., } \frac{18h}{b'} = 2 \;\; \therefore \;\; b' = 9h$$

i) Width and thickness of leaves

$$\text{Maximum equalized stress } \sigma = \frac{\alpha' F l}{i b' h^2} \quad \text{---- 20.18}$$

$$\text{i.e., } 400 = \frac{6\times 37500\times 500}{18\times 9h\times h^2}$$

∴ Thickness of each spring leaves h = 12 mm

Width of spring leaves b' = 9 h = 9 × 12 = 108 mm

ii) Initial space between full length and graduated leaves

$$\text{Camber } c = \frac{\beta' F l^3}{i b' E h^3} \qquad \text{----20.16}$$

$$= \frac{2\times 37500\times 500^3}{18\times 108\times 206.8\times 10^3\times 12^3} = 13.5\,\text{mm}$$

iii) Load on the band

$$\text{Load on the band } F_b = \frac{2F i_g i_f}{i(2i_g+3i_f)} = \frac{2\times 37500\times 15\times 3}{18[2\times 15+3\times 3]} = 4807.7\text{ N}$$

Example 3.30

A truck spring has 12 number of leaves, 2 of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5400 N with a permissible stress of 0.28 GPa. The ratio of total depth to width of spring is 3 and modulus of elasticity = 210 GPa. Determine (i) Thickness and width of steel spring (ii) Maximum deflection.

Data :

$i = 12$; $i_f = 2 = i'$ ∴ $i_g = 10$; $L = 1.05$ m $= 1050$ mm; $l_b = 85$ mm; $2F = 5400$ N ∴ $F = 2700$ N

$\sigma_f = 0.28$ GPa $= 280$ N/mm² (∵ not equally stressed); $\frac{ih}{b'} = 3$ i.e., $\frac{12h}{b'} = 3$ ∴ $b' = 4h$

$E = 210$ GPa $= 210 \times 10^3$ N/mm²

Solution :

i) Thickness and width of steel spring

$$\text{Effective length } l = \frac{L-l_b}{2} = \frac{1050-85}{2} = 482.5\text{ mm}$$

From table 20.3

$$\alpha = \beta = \frac{12}{2+r} = \frac{12}{2+\frac{i'}{i}} = \frac{12}{2+\frac{2}{12}} = 5.538$$

$$\text{Maximum stress } \sigma_f = \frac{1.5\alpha F l}{i b' h^2} \qquad \text{---- 20.14}$$

$$\text{i.e., } 280 = \frac{1.5\times 5.538\times 2700\times 482.5}{12\times(4h)(h^2)}$$

$$\therefore\ h = 9.3\text{ mm}$$

take, thickness of each leaf spring h = 9.5 mm

width of spring leaves b' = 4 h = 4 × 9.5 = 38 mm

ii) Maximum deflection

$$y = \frac{\beta F l^3}{E i b' h^3} = \frac{5.538 \times 2700 \times 482.5^3}{210 \times 10^3 \times 12 \times 38 \times 9.5^3} = 20.46 \text{ mm} \qquad \text{----}20.15$$

Example 3.31

Design a leaf spring for the following specifications for a truck. Total load = 120 kN. Number of springs = 4. Material for the spring is chrome-vanadium steel. Permissible stress is 0.55 GPa. Span of spring = 1100 mm. Width of central band = 100 mm and allowable deflection = 80 mm. Number of full length leaves are 2 and graduated leaves 6.

Data :

Total load W = 120 kN = 120 × 10^3 N; Number of springs = 4; Spring material – Chrome vanadium steel; σ_f = 0.55 GPa = 550 N/mm² (∵ not equally stressed); L = 1100 mm; l_b = 100 mm; y = 80 mm; i_f = i' = 2; i_g = 8

Solution :

$$\text{Effective length } l = \frac{L - l_b}{2} = \frac{1100 - 100}{2} = 500 \text{ mm}$$

$$\text{Total number of leaves } i = 2 + 6 = 8$$

$$\text{Load on each spring } 2F = \frac{\text{Total load}}{\text{Number of springs}} = \frac{120 \times 10^3}{4} = 30 \times 10^3 \text{ N}$$

$$\therefore F = 15000 \text{ N}$$

From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB) for Chrome Vanadium Steel

$$E = 206.92 \text{ GPa} = 206.92 \times 10^3 \text{ N/mm}^2$$

From Table 20.3 (Old DDHB) or Table 20.7 (New DDHB)

$$\alpha = \beta = \frac{12}{2 + r} = \frac{12}{2 + \frac{i'}{i}} = \frac{12}{2 + \frac{2}{8}} = 5.333$$

$$\alpha' = 6 \,;\, \beta' = 2$$

i) Width and thickness of spring leaves

$$\text{Permissible stress } \sigma_f = \frac{1.5 \alpha F l}{i b' h^2} \qquad \text{----}20.14$$

$$\text{i.e., } 550 = \frac{1.5 \times 5.333 \times 15000 \times 500}{8 \times b' h^2}$$

$$b'h^2 = 13636.36364 \qquad \text{----(i)}$$

$$\text{Maximum deflection } y = \frac{\beta F l^3}{E i b' h^3} \qquad \text{----}20.15$$

$$\text{i.e., } 80 = \frac{5.333 \times 15000 \times 500^3}{206.92 \times 10^3 \times 8 \times b' h^3}$$

$$\therefore b'h^3 = 75512.27528 \qquad \text{----(ii)}$$

Equation (ii) divided by (i) gives

$$\frac{b' h^3}{b' h^2} = h = \frac{75512.27528}{13636.36364} = 5.537 \text{ mm}$$

take, thickness of spring leaves h = 6 mm

Substituting in equation (i) b' = 378.788 mm

Substituting in equation (ii) b' = 349.6 mm

Select the bigger value as the permissible value

$\therefore$ Width of spring b' = 378.788 mm

ii) Initial space between full length and graduated leaves

$$\text{Camber } c = \frac{\beta' F l^3}{i b' E h^3} = \frac{2 \times 15000 \times 500^3}{8 \times 378.788 \times 206.92 \times 10^3 \times 6^3} = 27.688 \text{ mm}$$

iii) Load on the central band

$$F_b = \frac{2 F i_g i_f}{i(2 i_g + 3 i_f)} = \frac{2 \times 15000 \times 6 \times 2}{8[2 \times 6 + 3 \times 2]} = 2500 \text{ N}$$

Example 3.32

A multi-leaf spring with camber is fitted to the chassis of an automobile over a span of 1.2 m to absorb shocks due to a maximum load of 20 kN. The spring material can sustain a maximum stress of 0.4 GPa. All the leaves of the spring were to receive the same stress. The spring is required at least 2 full length leaves out of 8 leaves. The leaves are assembled with bolts over a span of 150 mm width at the middle. Design the spring for a maximum deflection of 50 mm.

Data :

L = 1.2 m = 1200 mm; 2 F = 20 kN = 20000 N $\therefore$ F = 10000 N; σ = 0.4 GPa = 400 N/mm^2 ($\because$ Equally stressed); $i_f = i' = 2$; i = 8 $\therefore$ $i_g = 6$; l_b = 150 mm; y = 50 mm.

Solution :

$$\text{Effective length } l = \frac{L - l_b}{2} = \frac{1200 - 150}{2} = 525 \text{ mm}$$

Assume E = 206.92 GPa = 206.92 × 10^3 N/mm^2

From table 20.3

$$\alpha = \beta = \frac{12}{2 + r} = \frac{12}{2 + \frac{i'}{i}} = \frac{12}{2 + \frac{2}{8}} = 5.333$$

$$\alpha' = 6; \quad \beta' = 2$$

i) Width and thickness of spring leaves

$$\text{Maximum equalised stress } \sigma = \frac{\alpha' F l}{i b' h^2} \quad \text{---- 20.18}$$

$$\text{i.e., } 400 = \frac{6 \times 10000 \times 525}{8 \times b' h^2}$$

$$\therefore b'h^2 = 9843.75 \quad \text{---- (i)}$$

$$\text{Maximum deflection } y = \frac{\beta F l^3}{Eib' h^3} \quad \text{----20.15}$$

$$\text{i.e., } 50 = \frac{5.333 \times 10000 \times 525^3}{206.92 \times 10^3 \times 8 \times b' h^3}$$

$$b'h^3 = 93242.5575 \quad \text{----(ii)}$$

Equation (ii) divided by (i) gives

$$\frac{b' h^3}{b' h^2} = h = \frac{93242.5575}{9843.75} = 9.47 \text{ mm}$$

take thickness of spring h = 9.5 mm

Substituting in equation (i) b' = 109 mm

Substituting in equation (ii) b' = 108.754 mm

Select the bigger value as the permissible value

$\therefore$ width of spring leaves b' = 109mm

ii) Initial space between full length and graduated leaves

$$\text{Camber } c = \frac{\beta' F l^3}{ib' Eh^3} \quad \text{---- 20.16}$$

$$= \frac{2 \times 10000 \times 525^3}{8 \times 109 \times 206.92 \times 10^3 \times 9.5^3} = 18.7 \text{ mm}$$

iii) Load on the central band

$$F_h = \frac{2Fi_g i_f}{i(2i_g + 3i_f)} = \frac{2 \times 10000 \times 6 \times 2}{8[2 \times 6 + 3 \times 2]} = 1666.667 \text{ N}$$

Example 3.33

A car weighing 9000 N empty has a seating capacity of 5 passengers each weighing 600 N. It has a wheel base of 2000 mm has its cg at 1100 mm behind the front axle. The car is to be supported on 4 similar long semi-elliptical carriage springs of each 840 mm between shackle pins. Design a suitable spring using a factor of safety of 1.5 on the proof stress of 0.84 GPa. The static loads are to be multiplied by a load factor of 2.5 to allow the impact load. The maximum deflection of the leaves equal to 50 mm. Number of full length leaves are 2 and graduated leaves 8. Take E = 210 GPa.

Solution :

Total load W = [Weight of car + weight of each passenger × Number of passengers] load factor = [9000 + 5 × 600] 2.5 = 30000 N

Since the total load will concentrate at the cg, taking moments about front axle,

$$R_A \times 2000 = 30000 \times 1100$$

$$\therefore R_A = 16500 \text{ N} = \text{Total load on rear axle}$$

$$\therefore \text{Total load on front axle } F_A = 30000 - 16500 = 13500 \text{ N}$$

Since the rear axle springs are heavily loaded, the design should be based on rear axle spring.

$\therefore$ Load on each rear axle spring $2F = R_A/2 = 16500/2 = 8250$ N

$$\therefore \; F = \frac{8250}{2} = 4125\,\text{N}$$

Effective length $l = \frac{l - l_b}{2} = \frac{840 - 0}{2} = 420$ mm

[∵ No central band]

Maximum permissible stress $\sigma_f = \frac{\sigma_y}{FOS} = \frac{840}{1.5} = 560$ N/mm²

Total number of leaves $i = i_f + i_g = 2 + 8 = 10$

From Table 20.3 (Old DDHB) or Table 20.7 (New DDHB)

$$\alpha = \beta = \frac{12}{2+r} = \frac{12}{2+\frac{i'}{i}} = \frac{12}{2+\frac{2}{10}} = 5.4545$$

$$\alpha' = 6; \; \beta' = 2$$

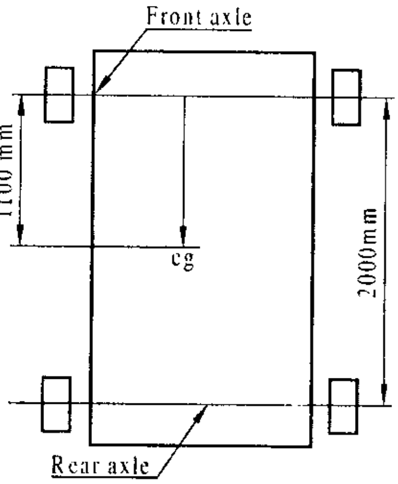


Fig. 3.19

i) Width and thickness of spring leaves

Maximum stress $\sigma_f = \frac{1.5\alpha F l}{i b' h^2}$ ---- 20.14

$$\text{i.e., } 560 = \frac{1.5 \times 5.4545 \times 4125 \times 420}{10 \times b' h^2}$$

$\therefore \; b' h^2 = 2531.25$ ---- (i)

Maximum deflection $y = \frac{\beta F l^3}{E i b' h^3}$ ---- 20.15

$$\text{i.e., } 50 = \frac{5.4545 \times 4125 \times 420^3}{210 \times 10^3 \times 10 \times b' h^3}$$

$\therefore \; b' h^3 = 15876$ ---- (ii)

Equation (ii) dividing by (i) gives

$$\frac{b' h^3}{b' h^2} = h = \frac{15876}{2531.25} = 6.275 \text{ mm}$$

take, thickness of spring leaves h = 6.5 mm

Substituting in equation (i) b' = 59.91 mm

Substituting in equation (ii) b' = 57.8 mm

Select the bigger value as the permissible value

∴ Width of spring leaves b' = 59.91 mm

ii) Initial space between full length and graduated leaves

Camber $c = \frac{\beta' F l^3}{i b' E h^3}$ ---- 20.16

$$= \frac{2 \times 4125 \times 420^3}{10 \times 59.91 \times 210 \times 10^3 \times 6.5^3} = 17.69 \text{ mm}$$

iii) Load on the central bolt

$$F_b = \frac{2Fi_g i_f}{i(2i_g + 3i_f)} = \frac{2 \times 4125 \times 8 \times 2}{10[2 \times 8 + 3 \times 2]} = 600 \text{ N}$$

Example 3.34

A truck spring has 10 leaves of graduated length. The spring supports 1060 mm apart and central band is 80 mm. The central load is to be 5400 N with a permissible stress of 0.28 GPa. The spring should have a ratio of total depth to width is about 2.5. Determine the width and thickness of spring plate and deflection when loaded. To what radius should the leaves be bent initially for the spring to be flat under the given load. Take E = 210 GPa.

Data :

$i = 10$; $L = 1060$ mm; $l_b = 80$ mm; $2F = 5400$ N $\therefore$ $F = 2700$ N; $\sigma_f = 0.28$ GPa $= 280$ N/mm²

$\frac{ih}{b'} = 2.5$ i.e., $\frac{10h}{b'} = 2.5$ $\therefore$ $b' = 4h$; $E = 210$ GPa $= 210 \times 10^3$ N/mm².

Solution :

Effective length $l = \frac{L - l_b}{2} = \frac{1060 - 80}{2} = 490$ mm

Since all the leaves are graduated, only one will extend to full length

$$\therefore \quad i_f = i' = 1 \text{ and } i_g = 9$$

From table 20.3

$$\alpha = \beta = \frac{12}{2 + r} = \frac{12}{2 + \frac{i'}{i}} = \frac{12}{2 + \frac{1}{10}} = 5.7143$$

$$\alpha' = 6 \text{ and } \beta' = 2$$

i) Width and thickness of spring leaves

$$\text{Maximum stress } \sigma_f = \frac{1.5 \alpha F l}{i b' h^2} \quad \text{---- 20.14}$$

$$\text{i.e., } 280 = \frac{1.5 \times 5.7143 \times 2700 \times 490}{10 \times 4h \times h^2}$$

$$\therefore \quad h = 10 \text{ mm} = \text{thickness of spring leaves.}$$

$$\text{Width of spring leaves } b' = 4h = 4 \times 10 = 40 \text{ mm}$$

ii) Maximum deflection

$$y = \frac{\beta F l^3}{E i b' h^3} \quad \text{---- 20.15}$$

$$= \frac{5.7143 \times 2700 \times 490^3}{210 \times 10^3 \times 10 \times 40 \times 10^3} = 21.609 \text{ mm}$$

iii) Initial radius of the leaves (R)

Since the spring has to be flat under the load, Camber c = Deflection $y = 21.609$ mm

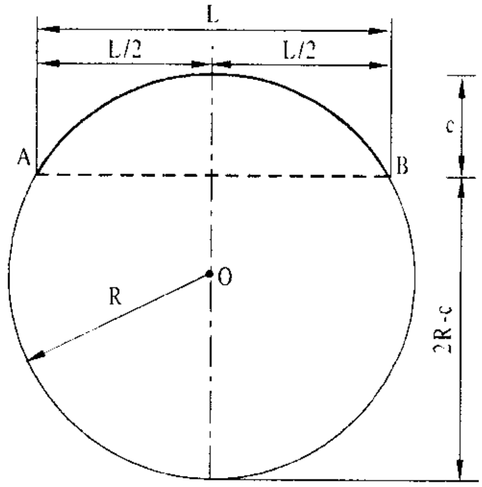


Fig. 3.20

From the geometry,

$$c\,(2\,R - c) = \frac{L}{2}.\frac{L}{2}$$

$$\text{i.e.,}\quad 2\,Rc - c^2 = \frac{L^2}{4}$$

$$\therefore\quad R = \frac{\frac{L^2}{4} + c^2}{2c} = \frac{\frac{1060^2}{4} + 21.609^2}{2 \times 21.609} = 6510.41 \text{ mm}$$

Example 3.35

Design a semi-elliptical laminated spring 1060 mm between centre of hooks held together at centre by 60 mm wide band and carrying a total load of 5000 N permissible stress for the spring material is 500 N/mm². Calculate the number of leaves, width and thickness of the leaves if the deflection is not to exceed 75 mm when the leaves are unstressed initially. Assume b' = 20 h and at least 2 leaves must be full length.

Data: $L = 1060$ mm; $l_b = 60$ mm; $2F = 5000$ N $\therefore$ $F = 2500$ N; $\sigma_f = 500$ N/mm²; $y = 75$ mm; $b' = 20\,h$; $i_f = i' = 2$;

Solution :

$$\text{Effective length}\quad l = \frac{L - l_b}{2} = \frac{1060 - 60}{2} = 500 \text{ mm}$$

$$\text{Maximum stress}\quad \sigma_f = \frac{1.5\,\alpha\,F l}{i b' h^2} \qquad \text{---- 20.14}$$

$$\text{i.e.,}\quad 500 = \frac{1.5\alpha \times 2500 \times 500}{i b' h^2}$$

$$\therefore\quad i b' h^2 = 3750\,\alpha \qquad \text{---- (i)}$$

$$\text{Maximum deflection } y = \frac{\beta F l^3}{E i b' h^3} \quad \text{Assume } E = 206.92 \text{ GPa} = 206.92 \times 10^3 \text{ N/mm}^2$$

$$\text{i.e., } 75 = \frac{\beta \times 2500 \times 500^3}{206.92 \times 10^3 \, i b' h^3}$$

$$\therefore \; i b' h^3 = 20136.6\beta \qquad \text{----(ii)}$$

Equation (ii) divided by (i) gives

$$\frac{i b' h^3}{i b' h^2} = h = \frac{20136.6\beta}{3750\alpha} = 5.37 \quad (\because \; \alpha = \beta)$$

take, thickness of spring leaves $h = 5.5$ mm

Width of spring leaves $b' = 20\,h = 20 \times 5.5 = 110$ mm

From equation (i)

$$i\, b'\, h^2 = 3750\,\alpha$$

$$\text{i.e., } i \times 110 \times 5.5^2 = 3750\left(\frac{12}{2+r}\right) \quad \text{[From Table 20.3(Old DDHB) or Table 20.7 (New DDHB)]}$$

$$= 3750\left(\frac{12}{2+\frac{i'}{i}}\right) = 3750\left[\frac{12}{2+\frac{2}{i}}\right]$$

$$\therefore \; i\left(2+\frac{2}{i}\right) = 13.52$$

$$\text{i.e., } 2i + 2 = 13.52$$

$$\therefore \; i = 5.76 \cong 6$$

$\therefore$ Total number of leaves $i = 6$

Number of full length leaves $i_f = 2$

Number of graduated leaves $i_g = 4$

3.19 HELICAL TORSION SPRING

A helical torsion spring is as shown in Fig. 3.21. It is used in door hinges, levers, pawl ratchets and various other electrical devices where torque is required. It is wound similar to extension or compression springs but have the ends shaped to transmit torque. The helical torsion spring resists the bending moment which tends to wind up the spring. The primary stresses in this spring are flexural in contrast with torsional shear stresses in compression or extension springs. The torsional moment on a spring produces a bending stress in the wire, the bending moment on the wire being numerically equal to torsional moment. In addition to this stress, there is direct tensile or compressive stress due to the force F that is tangential to the coil. Each individual section of the torsion spring is considered as a curved beam. Therefore using curved beam principle bending stress in torsion spring considering stress concentration factor is $\frac{k' M_b y}{I} = \frac{k' M_t}{Z}$

Maximum stress in torsion spring $\sigma = \frac{M_t}{Z} + \frac{F}{A}$ = Bending stress + direct stress ---- 20.87

The stress in torsion spring taking into consideration the correction factor k'

$$= \frac{k' M_t}{Z} + \frac{2M_t}{DA} \quad \text{---- 20.88}$$

Stress in a round wire spring $\sigma = \frac{8M_t(4k'D + d)}{\pi d^3 D}$ where $k' = k_l$ from figure 20.5 [DDHB]

---- 20.90 (Old DDHB) or 20.89a (New DDHB)

Deflection in torsion spring $y = \frac{M_t L D}{2EI}$ ---- 20.89

where M_t = Twisting moment

c = Spring Index $= \frac{D}{d}$

F = Force applied

i = Number of active turns

D = Mean diameter of coil

d = Wire diameter

L = Length of wire = πDi

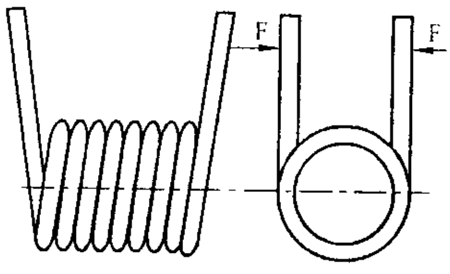


Fig. 3.21 : Helical torsion spring

A = Cross-sectional area of wire $= \frac{\pi}{4} d^2$

Z = Section modulus $= \frac{\pi d^3}{32}$

I = Moment of inertia $= \frac{\pi d^4}{64}$

E = Modulus of elasticity

Example 3.36

A helical torsional spring of mean diameter 50 mm is made of a round wire of 5 mm diameter. If a torque of 5 Nm is applied on this spring, find the bending stress, maximum stress and deflection of the spring in degrees. Modulus of elasticity = 200 GPa and number of effective turns 10.

Data :

D = 50 mm; d = 5 mm; M_t = 5 Nm = 5000 Nmm; E = 200 GPa = 200 × 10^3 N/mm²; i = 10

Solution :

Spring index $c = \frac{D}{d} = \frac{50}{5} = 10$

From Fig. 20.5 (DDHB) when c = 10, stress factor $k' = k_l = 1.08$

$$\text{Bending stress induced } \sigma_b = \frac{k' M_t}{Z} = \frac{k' M_t}{\frac{\pi}{32} d^3} = \frac{1.08 \times 5000}{\frac{\pi}{32} \times 5^3} = 440.032 \text{ N/mm}^2$$

$$\text{Maximum stress } \sigma = \frac{k' M_t}{Z} + \frac{2M_t}{DA} = 440.032 + \frac{2 \times 5000}{50 \times \frac{\pi}{4} \times 5^2} = 450.218 \text{ N/mm}^2$$

$$\text{Axial deflection } y = \frac{M_t LD}{2EI} \qquad \text{---- 20.89}$$

$$= \frac{M_t (\pi D_i) D}{2E \times \frac{\pi}{64} d^4} = \frac{5000 (\pi \times 50 \times 10)(50)}{2 \times 200 \times 10^3 \times \frac{\pi}{64} \times 5^4} = 32 \text{ mm}$$

$$\text{Also } y = \theta \cdot \frac{D}{2}$$

$$\text{i.e., } 32 = \theta \times \frac{50}{2}$$

$$\therefore \theta = 1.28 \text{ radian} = 1.28 \times \frac{180}{\pi} = 73.3386^\circ$$

NOTE : Since it is a round c/s wire, to find the maximum stress we can also use the Formulae 20.90.

3.20 BELLEVILLE SPRINGS

Disc springs, also called Belleville springs are used where high capacity compression springs must fit into small spaces. Each spring consists of several annular discs that are dished to a conical shape as shown in Fig. 3.22 a. They are stacked up one on top of another as shown in Fig. 3.22 b. When the load is applied, the discs tend to flatten out, and this elastic deformation constitutes the spring action. In a safety valve the disc springs are used.

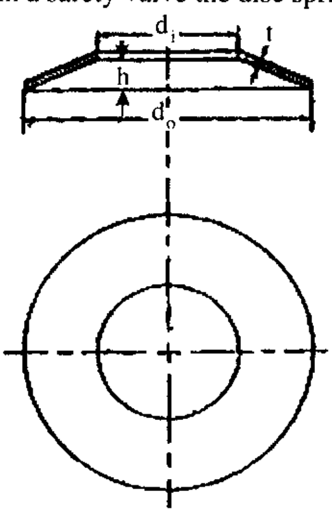


Fig. 3.22 a [DDHB Fig. 20.2] : Disk spring

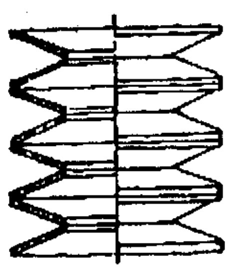


Fig. 3.22 b : Nest of disc spring

The relation between the load F and the axial

deflection y of each disc $F = \frac{4Ey}{(1-\upsilon^2)Md_o^2}\left[(h-y)\left(h-\frac{y}{2}\right)t+t^3\right]$ ---- 20.19

Maximum stress induced at the inner edge

$$\sigma_i = \frac{4Ey}{(1-\upsilon^2)Md_o^2}\left[C_1\left(h-\frac{y}{2}\right)+C_2t\right] \quad \text{---- 20.20}$$

Maximum stress induced at the outer edge

$$\sigma_o = \frac{4Ey}{(1-\upsilon^2)Md_o^2}\left[C_1\left(h-\frac{y}{2}\right)-C_2t\right] \quad \text{---- 20.21}$$

where C_1 and C_2 are constants from Fig. 20.3 (DDHB)

Let t = thickness

h = height of spring

M = constant from Fig. 20.3 (DDHB)

F = axial force

d_o = outer diameter

d_i = inner diameter

υ = Poisson's ratio

E = modulus of elasticity

y = axial deflection

The ratio $\frac{d_o}{d_i}$ should lie between 1.5 and 3.

Example 3.37

A belleville spring is made of 3 mm sheet steel with an outside diameter of 125 mm and an inside diameter of 50 mm. The spring is dished 5 mm. The maximum stress is to be 500 MPa. Determine

(i) Safe load carried by the spring

(ii) Deflection at this load

(iii) Stress produced at the outer edge

(iv) Load for flattening the spring

Assume Poisson's ratio = 0.3 and E = 200 GPa

Data :

t = 3 mm; d_o = 125 m; d_i = 50 mm; h = 5 mm; σ_n = 500 MPa = 500 N/mm^2; υ = 0.3;

E = 200 GPa = 200 × 10^3 N/mm^2

Solution :

From Fig. 20.3 (DDHB) for $\frac{d_o}{d_i} = \frac{125}{50} = 2.5$

$M = 0.75;\ C_1 = 1.325;\ C_2 = 1.54$

i) Deflection

Stress at the inner edge $\sigma_{ri} = \frac{4Ey}{(1-\upsilon^2)d_o^2 M}\left[\left(h-\frac{y}{2}\right)C_1 + C_2 t\right]$ ---- 20.20

i.e., $500 = \frac{4\times 200\times 10^3 \times y}{(1-0.3^2)0.75\times 125^2}\left[1.325\left(5-\frac{y}{2}\right)+1.54\times 3\right]$

$6.665 = y\,[6.625 - 0.6625\,y + 4.62]$

$= 11.245\,y - 0.6625\,y^2$

i.e., $+0.6625\,y^2 - 11.245\,y + 6.665 = 0$

$\therefore\; y = +\frac{11.245 \pm \sqrt{11.245^2 - 4\times 0.6625\times 6.665}}{2\times 0.6625}$

$= 16.36$ mm or 0.615 mm

$\therefore$ Deflection $y = 0.615$ mm $(\because\; y < h)$

ii) Safe load

$F = \frac{4Ey}{(1-\upsilon^2)Md_o^2}\,[(h-y)(h-y/2)\,t + t^3]$ ---- 20.19

$= \frac{4\times 200\times 10^3\times 0.615}{(1-0.3^2)0.75\times 125^2}\left[(5-0.615)\left(5-\frac{0.615}{2}\right)3+3^3\right]$

$= 4093.66$ N

iii) Stress at the outer edge

$\sigma_o = \frac{4Ey}{(1-\upsilon^2)Md_o^2}\,[C_1(h-y/2) - C_2 t]$ ---- 20.21

$= \frac{4\times 200\times 10^3\times 0.615}{(1-0.3^2)0.75\times 125^2}\left[1.325\left(5-\frac{0.615}{2}\right)-1.54\times 3\right]$

$= 73.706$ N/mm^2

iv) Load for flattering the spring

When $y = h$, the spring will become flat.

$\therefore\; F = \frac{4Eht^3}{(1-\upsilon^2)Md_o^2} = \frac{4\times 200\times 10^3\times 5\times 3^3}{(1-0.3^2)0.75\times 125^2} = 10127.5\,\text{N}$

3.21 RUBBER SPRINGS

Rubber springs have good damping properties and hence it is used in vibration loading. Rubber is usually bonded to metal plates and can be used in tension, compression and shear. In shear rubber displays maximum elastic properties and in compression it displays maximum stiffness. At high temperature and in oil rubber cannot be used.

3.22 COMBINATION OF SPRINGS

Example 3.38

A 100 mm outside diameter steel coil spring having 10 active coils of 12.5 mm diameter wire is in contact with a 600 mm long steel cantilever spring having 5 graduated leaves 100 mm wide and 10 mm thick as shown in Fig. 3.23.

(i) What force F is gradually applied to the top of the coil spring will cause the cantilever to deflect by 50 mm.

(ii) What is the bending stress in cantilever beam.

(iii) What is the shear stress in coil spring

(iv) What energies stored by each spring. Take 210 GPa and G = 84 GPa

Data :

Cantilever	Coil
$i = 5$	$i = 10$
$l = 600$ mm	$d = 12.5$ mm
$b' = 100$ mm	$D_o = 100$ mm $\therefore D = 100 - 12.5 = 87.5$ mm
$y = 50$ mm	$G = 84$ GPa $= 84 \times 10^3$ N/mm^2
$E = 210$ GPa $= 210 \times 10^3$ N/mm^2	
$h = 10$ mm	

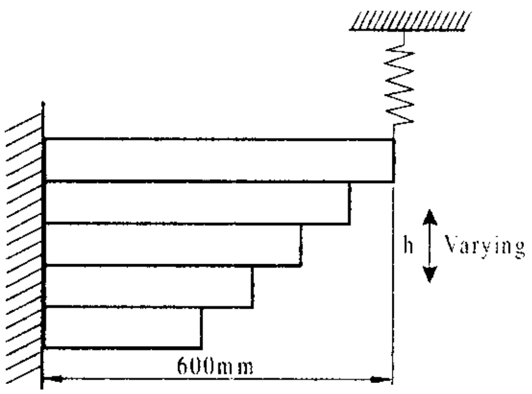


Fig. 3.23

Solution :

$$c = \frac{D}{d} = \frac{87.5}{12.5} = 7$$

Since the coil spring is on the top of the cantilever, load on cantilever = load on coil spring

i.e., $F_1 = F_2$

From Table 20.1 for constant width varying depth

$c_1 = 6$ and $c_2 = 8$

i) Cantilever spring

$$\text{Deflection } y = \frac{c_2 F l^3}{E i b' h^3} \qquad \text{---- 20.5}$$

$$\text{i.e., } 50 = \frac{8 \times F_1 \times 600^3}{210 \times 10^3 \times 5 \times 100 \times 10^3}$$

$\therefore$ Load applied on the top of the coil spring $F_1 = 3038.2$ N

ii) Bending stress in cantilever spring

$$\sigma = \frac{c_1 F l}{i b' h^2} = 6 \times \frac{3038.2 \times 600}{5 \times 100 \times 10^2} = 218.75 \text{ N/mm}^2$$

iii) Shear stress in coil spring

$$\tau = \frac{8FDk}{\pi d^3} \; ; \; k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4 \times 7 - 1}{4 \times 7 - 4} + \frac{0.615}{7} = 1.2128$$

$$= \frac{8 \times 3038.2 \times 87.5 \times 1.2128}{\pi \times 12.5^3} = 420 \text{ N/mm}^2$$

iv) Energy stored

Energy stored in the cantilever spring $U_1 = \frac{1}{2} F_1 y_1$

$$= \frac{1}{2} \times 3038.2 \times 50 = 75955 \text{ Nmm} \approx 76 \text{ Nm}$$

Energy stored in coil spring $U_2 = \frac{1}{2} F_2 y_2$ where

$$y_2 = \frac{8FD^3 i}{Gd^4} = \frac{8 \times 3038.2 \times 87.5^3 \times 10}{84 \times 10^3 \times 12.5^4} = 79.4 \text{ mm}$$

$$\therefore \; U_2 = \frac{1}{2} \times 3038.2 \times 79.4 = 120616.54 \text{ N/mm} \approx 120.62 \text{ Nm}$$

Example 3.39

A semi elliptical leaf spring has a span length of 1.8 m. The spring seat is midway between the shackles, carries a helical spring upon which is imposed an impact equal to 2500 Nm of energy. The laminated spring is composed of 10 graduated and 2 full length leaves each 5 mm thick and 50 mm wide. The coil spring comprises of 6 effective turns, 15 mm wire diameter and has a mean diameter of 100 mm. Calculate the maximum stress induced in each spring. Take E = 210 GPa and G = 84 GPa.

Data :

Leaf spring	*Coil spring*
$i_g = 10$	$i = 6$
$i_f = 2$	$d = 15$ mm
$h = 5$ mm	$D = 100$ mm
$b' = 50$ mm	$G = 84$ GPa $= 84 \times 10^3$ N/mm^2
$L = 1800$ mm	
$l_b = 0$	
$U = 2500$ Nm $= 25 \times 10^5$ Nmm	
$E = 210$ GPa $= 210 \times 10^3$ N/mm^2	

Solution :

Coil spring

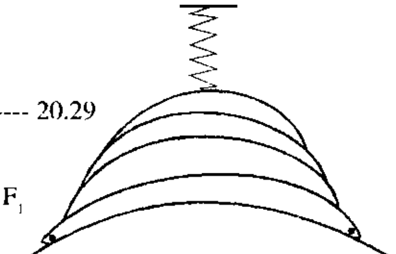


Fig. 3.24

$$\text{Maximum deflection}\quad y_1 = \frac{8F_1D^3i}{Gd^4} \qquad \text{---- 20.29}$$

$$= \frac{8F_1 \times 100^3 \times 6}{84 \times 10^3 \times 15^4} = 0.0113\ F_1$$

$$\text{Now, energy stored in the spring } U = \frac{1}{2} F_1 y_1$$

$$\text{i.e.,}\quad 25 \times 10^5 = \frac{1}{2} \times F_1 \times 0.0113\ F_1$$

$$\therefore\ F_1 = 21046.8\text{ N}$$

$$y_1 = 237.565\text{ mm}$$

$$c = \frac{D}{d} = \frac{100}{15} = 6.667$$

$$\text{Shear stress}\quad \tau = \frac{8F_1Dk}{\pi d^3}$$

$$\therefore\ k = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

$$= \frac{8 \times 21046.8 \times 100 \times 1.2245}{\pi \times 15^3}$$

$$= \frac{4 \times 6.667 - 1}{4 \times 6.667 - 4} + \frac{0.615}{6.667} = 1.2245$$

$$= 1944.5\text{ N/mm}^2$$

Leaf spring

$$i = i_f + i_g = 2 + 10 = 12$$

$$\text{Effective length}\quad l = \frac{L - l_b}{2} = \frac{1800 - 0}{2} = 900\text{ mm}$$

From Table 20.3 (Old DDHB) or Table 20.7 (New DDHB)

$$\alpha = \beta = \frac{12}{2+r} = \frac{12}{2+\frac{i'}{i}} = \frac{12}{2+\frac{2}{12}} = 5.538$$

$$\text{Maximum deflection}\quad y_2 = \frac{\beta F_2 l^3}{Eib'h^3}$$

$$= \frac{5.538 \times F_2 \times 900^2}{210 \times 10^3 \times 12 \times 50 \times 5^3} = 0.25635\ F_2$$

$$\text{But}\quad U_2 = \frac{1}{2} F_2 y_2$$

$$\text{i.e.,}\quad 25 \times 10^5 = \frac{1}{2} F_2 \times 0.25635\ F_2$$

$$\therefore\ F_2 = 4416.4\text{ N}$$

$$y_2 = 1132.14\text{ mm}$$

$$\text{Maximum stress}\quad \sigma_f = \frac{1.5\alpha F_2 l}{ib'h^2} = \frac{1.5 \times 5.538 \times 4416.4 \times 900}{12 \times 50 \times 5^2}$$

$$= 2201.2\text{ N/mm}^2$$

3.23 SPRINGS SUBJECTED TO FATIGUE LOADING

Example 3.40

A spring is subjected to a load varying from 400 N to 1000 N is to be made of tempered steel cold wound wire. Determine the diameter of wire and mean coil diameter of spring for a factor of safety of 1.5. Spring index 6. Torsional endurance limit is 400 N/mm².

Data :

$F_{max} = 1000$ N; $c = 6$; $F_{min} = 400$ N; $\tau_{-1} = 400$ N/mm²; FOS $n = 1.5$

Solution :

From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB) for oil tempered carbon wire $\tau_y = 0.55$ GPa $= 550$ N/mm²

First method [suggested by Wahl]

Variable stress amplitude $\tau_a = k_w \left(\frac{8D}{\pi d^3}\right)\left(\frac{F_{max} - F_{min}}{2}\right)$ Where $k_w = k_\tau k_c$ ---- 20.58

$$k_\tau = 1 + \frac{0.5}{c} = 1 + \frac{0.5}{6} = 1.0833$$

From Table 20.7 (Old DDHB) or Table 20.15 (New DDHB) for c = 6

$$k_c = 1.15$$

$$\therefore \; k_w = 1.0833 \times 1.15 = 1.2458 = k$$

$$c = \frac{D}{d} \quad \therefore \; D = cd = 6d$$

$$\therefore \; \tau_a = 1.2458\left(\frac{8 \times 6d}{\pi d^3}\right)\left[\frac{1000 - 400}{2}\right] = \frac{5710.32}{d^2}$$

Mean shear stress $\tau_m = k_\tau \left(\frac{8D}{\pi d^3}\right)\left(\frac{F_{max} + F_{min}}{2}\right)$ ---- 20.59

$$= 1.0833\left(\frac{8 \times 6d}{\pi d^3}\right)\left(\frac{1000 + 400}{2}\right) = \frac{11586.12}{d^2}$$

$$\text{Now} \quad n = \frac{\tau_y}{\tau_m - \tau_a + \frac{2\tau_a \tau_y}{\tau_{-1}}}$$

$$\text{i.e.,} \quad 1.5 = \frac{550}{\frac{11586.12}{d^2} - \frac{5710.32}{d^2} + \frac{2 \times 5710.32 \times 550}{d^2 \times 400}}$$

$$\therefore \; d = 7.67 \text{ mm}$$

From Table 20.12 (Old DDHB) std wire diameter d = 8 mm

Second method [suggested by Wahl]

$$\tau_{max} - \tau_{min} = \frac{k_w 8D\left[F_{max} - F_{min}\right]}{\pi d^3} = \frac{\tau_{-1}}{n}$$

i.e., $\frac{1.2458 \times 8 \times 6d[1000-400]}{\pi d^3} = \frac{400}{1.5}$

$\therefore$ d = 6.544, Hence 8 mm wire diameter is satisfactory

Also $\tau_{max} = \frac{k_\tau 8DF_{max}}{\pi d^3} = \frac{\tau_y}{n}$

i.e., $\frac{1.0833 \times 8 \times 6d \times 1000}{\pi d^3} = \frac{550}{1.5}$

$\therefore$ d = 6.72 mm, hence 8 mm wire diameter is satisfactory

$\therefore$ take wire diameter d = 8 mm

Mean diameter of coil D = $6d = 6 \times 8 = 48$ mm

Note :

For oil hardened and tempered steel wire

$\tau_y = 0.45\,\sigma_{ut}$ or $0.5\,\sigma_{ut}$

$\tau_{-1} = 0.22\,\sigma_{ut}$ or $\frac{530}{d^{0.2}}$ MN/m² where d in meters

---

3.24 TENSION SPRING

Example 3.41

Design a helical spring for a spring loaded safety valve for the following data:- Operating pressure = 1 MPa; Maximum pressure when the valve blows off = 1.075 MPa. Maximum lift of valve when pressure is 1.075 MPa = 6 mm. Diameter of valve seat = 100 mm. Maximum allowable shear stress = 0.4 GPa. Rigidity modulus = 86 GPa. Spring index = 5.5. Assume this to be a Ramsbottom safety valve. **(VTU, July 2007)**

Data :

P_1 = 1 MPa = 1 N/m²m; P_2 = 1.075 MPa = 1.075 N/mm²; y' = 6 mm

Diameter of valve seat = 100 mm; τ = 0.4 GPa = 400 MPa = 400 N/mm²

G = 86 GPa = 86000 MPa = 86000 N/mm²; c = 5.5

Solution :

As the spring is used in Ramsbottom safety valve, it will be under tension

Minimum force on the spring = Area of valve x $P_1 = \frac{\pi}{4} \times 100^2 \times 1 = 7853.98$ N = F_1

Maximum force on the spring = Area of valve x $P_2 = \frac{\pi}{4} \times 100^2 \times 1.075 = 8443.03$ N = F_2

Maximum deflection $y_2 = \frac{y' F_2}{F_2 - F_1} = \frac{6 \times 8443.03}{8443.03 - 7853.98}$ = **86 mm** ----20.31

Design the spring for maximum load and maximum deflection

1. Diameter of wire

Shear stress $\tau = \frac{8F_2Dk}{\pi d^3}$ ---- 20.22

Stress factor $k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times5.5-1}{4\times5.5-4} + \frac{0.615}{5.5} = 1.2785$ ---- 20.23

Spring index $c = \frac{D}{d}$ $\therefore$ $D = cd = 5.5\ d$

$$\therefore\ 400 = \frac{8\times8443.03\times5.5d\times1.2785}{\pi\times d^3}$$

$$d = 19.44 \text{ mm}$$

Select standard diameter of wire from Table 20.12 (Old DDHB)

$$\therefore\ \mathbf{d = 20\ mm}$$

2. Diameter of coil

Mean diameter of coil $D = 5.5\ d = 5.5 \times 20 = 110$ mm

Outer diameter of coil $D_o = D + d = 110 + 20 = 130$ mm

Inner diameter of coil $D_i = D - d = 110 - 20 = 90$ mm

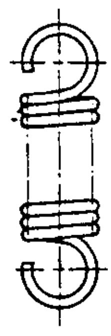


Fig. 3.25

3. Number of coils or turns

Maximum deflection $y_2 = \frac{8F_2D^3i}{d^4G}$ ---- 20.29

$$86 = \frac{8\times8443.03\times110^3\times i}{20^4\times86000}$$

i.e., $i = 13.16$

$\therefore$ Number of active turns **i = 14**

4. Free length

Since the spring having loops on both ends, the total number turns $i' = i + 1 = 14 + 1 = 15$

Least gap between two coils when the spring is in free state will be 1 mm

$\therefore$ Free length of spring $l_o = id + (i-1)\times1 = 14\times20 + (14-1)\ 1 = 293$ mm

5. Pitch

$$p = \frac{l_o}{i-1} = \frac{293}{14-1} = 22.54 \text{ mm}$$

6. Stiffness or Rate of spring

$$F_o = \frac{F}{y} = \frac{F_2}{y_2} = \frac{8443.03}{86} = 98.2 \text{ N/mm}$$

7. Total length of wire

$$l = \pi Di' = \pi\times110\times15 = 5183.628 \text{ mm}$$

Specifications :

(i) Wire diameter d = 20 mm

(ii) Mean coil diameter D = 110 mm

(iii) Free length $l_o = 293$ mm

(iv) Total number of turns $i' = 15$

(v) Style of ends – Full hook

(vi) Pitch $p = 22.54$ mm

(vii) Rate of springs $F_o = 98.2$ N/mm

Example 3.42

The spring used in an automobile engine has to exert 500N when the valve is closed and 600N when the valve is opened. The displacement of the valve is 5mm. The engine crankshaft rotates at 8000 rpm. Design the spring if permissible stress in the material of the spring is 300MPa. The ratio of mean coil diameter to the wire diameter is 6. The specific weight and the modulus of rigidity of the spring material ar 7.35×10^{-5} N/mm³ and 8×10^4 MPa, respectively. The ends of the spring are square and ground. Inspect the suitability of the spring for this engine. At what speed of the engine does the spring resonate? (VTU July 2006)

Data :

Minimum load $F_1 = 500N$

Maximum load $F_2 = 600N$

$y' = 5mm; \frac{D}{d} = c = 6$

Automobile Engine. $\therefore$ 4 stroke engine

Crank shaft speed $= 8000$ rpm

$\tau = 300 \text{ MPa} = 300 \text{ N/mm}^2$; $G = 8 \times 10^4 \text{ MPa} = 8 \times 10^4 \text{N/mm}^2$

Specific weight $\gamma = 7.35 \times 10^{-5} /\text{mm}^3$

Square and ground end.

Solution :

Design of spring

$$\text{Maximum deflection } y_2 = \frac{F_2 y'}{F_2 - F_1} = \frac{600 \times 5}{600 - 500} = 30\text{mm} \qquad \text{----20.31}$$

Design the spring for maximum load and maximum deflection

1. Diameter of wire

$$\text{Shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \qquad \text{----20.22}$$

$$\text{Wahl's stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525 \qquad \text{----20.23}$$

$$\text{Spring index } c = \frac{D}{d} = 6$$

$$\therefore D = 6d$$

$$\text{Hence } 300 = \frac{8 \times 600 \times 6d \times 1.2525}{\pi d^3}$$

$$\therefore \text{Wire diameter } d = 6.186\text{mm}$$

Select the standard wire diameter from Table 20.12 (Old DDHB)

$$\therefore d = 6.3\text{mm}$$

2. Diameter of coil

Mean diameter of coil $D = cd = 6 \times 6.3 = 37.8$ mm

Outer diameter of coil $D_o = D + d = 37.8 + 6.3 = 44.1$mm

Inner diameter of coil $D_i = D - d = 37.8 - 6.3 = 31.5$mm

3. Number of coils

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4G} \qquad \text{---20.29}$$

$$\text{i.e., } 30 = \frac{8 \times 600 \times 37.8^3 \times i}{6.3^4 \times 8 \times 10^4}$$

$$\text{i.e., } i = 14.583$$

$\therefore$ Number of active turns i = 15

4. Free length

$$l_o \geq (i + n)d + y + a \qquad \text{----20.53}$$

For square and ground end, number of additional coils n = 2

$$y = \text{Maximum deflection } y_2 = 30\text{ mm}$$

Clearance $a = 25\%$ of maximum deflection $= \frac{25}{100} \times 30 = 7.5$mm

$\therefore l_o \geq (15 + 2)\,6.3 + 30 + 7.5$

≥ 144.6mm

5. Pitch

$$P = \frac{l_o - 2d}{i}$$ From Table 20.6 ((Old DDHB) Table 20.14 (New DDHB)

$$= \frac{144.6 - 2 \times 6.3}{15} = 8.8\text{ mm}$$

6. Rate of spring or Stiffness of spring

$$F_o = \frac{F_2}{y_2} = \frac{600}{30} = 20\text{ N/mm} \qquad \text{----20.30}$$

7. Total length of wire

$$l = \pi Di' \text{ where } i' = i + n = 15 + 2 = 17$$

$$= \pi \times 37.8 \times 17 = 2018.8\text{mm}$$

Check for buckling or stability of the spring

$$\text{Ratio } \frac{l_o}{D} = \frac{144.6}{37.8} = 3.8254$$

From Figure 20.8 (DDHB) for $\frac{l_o}{D} = 3.8254$ and Built in ends factor $K_l = 0.63$

$\therefore$ Critical axial load that can cause bucking $F_{cr} = F_o K_l l_o$

$$= 20 \times 0.63 \times 144.6 = 1821.96N$$

Since the critical load 1821.96N is more than the maximum load of 600N, the spring will not buckle and hence safe.

Critical speed

The fundamental or critical frequency of the spring when both ends are fixed

$$f = \frac{1}{\pi}\sqrt{\frac{2k_o g}{W}} = \frac{1}{\pi}\sqrt{\frac{2F_o g}{W}} = 1.41\sqrt{\frac{F_o}{W}} \text{ Hz} \qquad \text{----20.76}$$

Weight of the active coil of a helical spring $W = \dfrac{\pi^2 d^2 D i \gamma}{4}$ ---- 20.47g

$$= \frac{\pi^2 \times 6.3^2 \times 37.8 \times 15 \times 7.35 \times 10^{-5}}{4} = 4.0812N$$

$$F_0 = 20 \text{ N/mm} = 20000 \text{ N/m}$$

$$\therefore f = 1.41\sqrt{\frac{20000}{4.0812}} = 98.705 \text{ Hz}$$

$$\therefore \text{Critical speed} = 98.705 \text{ rps} = 98.705 \times 60 = 5922.3 \text{ rpm}$$

Since the cam shaft speed is half of the engine speed for four stroke engine, the speed of the engine at which the spring resonate = 5922.3 x 2 = 11844.6 rpm. Lower engine speeds are not likely to set up surge action, because of the damping effect of the spring material between the impulses. [other dangerous speeds would be $\dfrac{11844.6}{2} = 5922.3$ rpm, $\dfrac{11844.6}{3} = 3948.2$ rpm and $\dfrac{11844.6}{4} = 2961.15$rpm]

The design of spring is safe, since the crank shaft speed is 8000 rpm which is far from 11844.6 rpm and 5922.3 rpm.

Example 3.43

Design a helical compression spring required for a spring loaded safety valve mounted on a pressure vessel. The spring is subjected to an initial compression of 50mm at the time of assembly and will open by 10mm when the pressure approaches 6MPa. The diameter of the valve is 25mm.

(VTU Jan/Feb, 2006)

Date :

Initial compression y_1 = 50mm; y' = 10mm

Maximum pressure P_2 = 6MPa = 6N/mm²

Diameter of the valve = 25mm

Solution :

Assume the spring material as Chrome - Vanadium alloy steel. Factor of safely = 1.5 and spring index c = 6. From Table 20.14 (Old DDHB) or Table 20.10 (New DDHB) for Chrome-Vanadium alloy steel.

$$\tau_y = 0.69 \text{ GPa} = 690 \text{ MPa} = 690 \text{ N/mm}^2$$

$$G = 79.34 \text{ GPa} = 79340 \text{ MPa} = 79340 \text{ N/mm}^2$$

$$\therefore \text{Allowable shear stress } \tau = \frac{\tau_y}{Fos} = \frac{690}{1.5} = 460 \text{N/mm}^2$$

Maximum deflection $y_2 = y_1 + y' = 50 + 10 = 60mm$ Refer Fig. 20.6 (DDHB)

Maximum Load $F_2 = P_2 \times$ Area of c/s of the valve

$$= 6 \times \frac{\pi}{4} \times 25^2 = 2945.243N$$

Design the spring for maximum load and maximum deflection

1. Diameter of spring wire

$$\text{Allowable shear stress } \tau = \frac{8F_2Dk}{\pi d^3} \quad \text{---- 20.22}$$

$$\text{Stress factor } k = \frac{4c-1}{4c-4} + \frac{0.615}{c} = \frac{4\times6-1}{4\times6-4} + \frac{0.615}{6} = 1.2525 \quad \text{---- 20.23}$$

$$\text{Spring index } c = \frac{D}{d} = 6$$

$$\therefore D = 6d$$

$$\therefore 460 = \frac{8 \times 2945.243 \times 6d \times 1.2525}{\pi d^3}$$

$$d = 11.07mm$$

Select the standard diameter of wire from Table 20.12 (Old DDHB)

$\therefore$ Diameter of spring wire d = 12mm

2. Diameter of coil

Mean diameter of coil $D = 6 \times 12 = 72mm$

Outer diameter of coil $D_o = D + d = 72 + 12 = 84mm$

Inner diameter of coil $D_i = D - d = 72 - 12 = 60mm$

3. Number of turns

$$\text{Maximum deflection } y_2 = \frac{8F_2D^3i}{d^4G} \quad \text{---- 20.29}$$

$$\text{ie, } 60 = \frac{8 \times 2945.243 \times 72^3 \times i}{12^4 \times 79340}$$

$$i = 11.22$$

$\therefore$ Number of active turns i = 12

Free length, pitch, Rate of spring, Total length of wire and Specification of spring wire are as calculated in other Examples.

Example 3.44

The spring loaded safety valve for a boiler is required to blow off at a pressure of 1 MPa. The diameter of the valve is 60mm and the maximum lift of the valve is 15mm. Design a suitable compression spring for the safety valve providing an initial compression of 30mm

Data :

Blow off pressure P_1 = 1 MPa; y' = 15mm

Initial compression y_1 = 30mm

Diameter of the valve = 60mm

Solution :

Load on the valve when it just begins to lift $F_1 = P_1 \times$ Area of c/s of the valve

$$= 1 \times \frac{\pi}{4} \times 60^2 = 2827.4334\ N$$

Maximum deflection $y_2 = y_1 + y' = 30 + 15 = 45mm$ Refer Fig. 20.6 (DDHB)

Also maximum deflection $y_2 = \frac{F_2 y'}{F_2 - F_1}$ 20.31 (DDHB)

ie, $45 = \frac{F_2 \times 15}{F_2 - 2827.4334}$

ie, $F_2 - 2827.4334 = 0.3333F_2$

$\therefore$ Maximum load $F_2 = 4241.15N$

Design the spring for maximum load and maximum deflection similar to the previous Example.

REVIEW QUESTIONS

1. Derive the equation for energy stored in a helical spring **VTU, Aug. 2001**
2. Derive an expression for the shear stress induced in a Helical compression spring, with usual notations. **VTU, Feb. 2002/July/Aug. 2002**
3. With usual notations, derive the equations for deflection and bending stresses induced in full length leaves and graduated leaves of a laminated spring. **VTU, July/Aug. 2003**
4. What are the requirements of spring materials? What are the important spring materials? **VTU, Jan./Feb. 2003**
5. Explain equalizing the stresses in leaf springs.
6. Define spring index, mean coil diameter and helix angle. **VTU, Dec. 2003**
7. Explain the importance of Wahl's factor in the design of Helical Springs. **BU, Mar/April 1999**
8. When do you recommend non circular wire springs? Why this cross section of the wire is not generally used? **BU, Aug. 1996**
9. What do you understand by surge in helical springs? How can it be eliminated. **BU, August 1997**
10. What is equalised stresses [Nipping] in spring leaves? Explain clearly. **VTU, August 2001 (IP/IM)**

EXERCISES

1. Two helical springs are nested and are arranged in a concentric manner, with one inside the other. Both the springs have the same free length and carry a total load of 5500 N. The details are as below :

	Outer Spring	inner Spring
i) No. active turns	8	12
ii) Wire dia	16 mm	12 mm
iii) mean coil dia	128 mm	84 mm

   Determine the max load carried by each spring; total deflection of springs and max stresses in each spring taking G = 81.4 GPa

2. Design a rectangular section helical spring to mount to a buffer to sustain a load of 30 kN. The initial compression in the spring upon mounting is 50 mm and further deflection up on load is limited to 100 mm. The spring is made of Z Nickle. The longer side of the section is made twice the shorter side and the spring is formed with longer side parallel to the axis. The clearance between each coil is to be 5 mm and the spring index is to be 10. Take G = 76 GPa and factor of safety 2.

3. A loaded narrow car of mass 1600 kg and moving at a velocity of 1.2 m/s is brought to rest by a bumper consisting of two helical steel springs of square section. The mean coil diameter of the spring is six times the side of the square. In bringing the car to rest the springs are compressed by 200 mm.

   The permissible shear stress is 400 MPa. Modulus of rigidity may be taken as 84 GPa.

   Determine the following :

   i) Mean load on each spring. ii) Side of the square section wire.

   iii) Mean coil diameter iv) Active number of coils

4. In a semielliptical laminated spring the given effective length is 900 mm, the total load is 3600 N. The maximum deflection is 75 mm and maximum stress due to bending is 360 N/mm^2. Decide the number of leaves and their breadth and thickness.
   Take E = 0.20 × 10^6 N/mm^2 for the material of the spring.

5. Design the leaf springs for the rear suspension of a heavy duty truck. The distance between the axles, front and rear is 6 m. The weight of the loaded truck is 2 MN and the center of the gravity of the truck lies at 2 m from the rear axle and at a height of 2 m from the ground. The number of full length is 2, the material for the spring is steel with a permissible strength of 250 MPa. The length of the spring between the eyes is 1600 mm and the central band is 100 mm wide. Determine the thickness and number of graduated leaves if the width of the leaf is not to exceed 75 mm. The maximum deflection of the spring is limited to 50 mm. Determine also the force required to pre stress the spring.

6. At the bottom of an elevator shaft a group of 12 identical closed coiled helical springs are set in parallel to absorb the shock caused by the falling of the cage in case of failure. The loaded cage weighs 20 kN. While the counter weight has a weight of 5 kN. If the loaded cage falls through a height of 3 m from rest, find the maximum stress induced in each spring if it is made of 40 mm diameter steel road. The spring index is 5 and the number of active turns in each spring is 4. Modulus of rigidity $G = 82.7 \times 10^3$ MN/m^2.

7. A laminated semi-elliptical leaf spring under a central.load of 10 kN is to have an effective length of 1 metre and is to deflect not more than 50 mm. The spring has 8 leaves, two of which are full length, and have been pre-stressed so that all leaves have the same stress after the full load has been applied. All the leaves have the same width and thickness. The maximum stress in the leaves is not to exceed 35 kg/mm^2.

   Determine :

   i) the width and thickness of leaves. ii) the central bolt load

   iii) the initial gap between the full length and graduated leaves before assembly.

8. a) Design a helical compression spring to sustain an axial load that fluctuates between 1.5 kN and 2 kN with an associated deflection of 15 mm during the fluctuation of load.

   b) An automotive leaf spring is to be designed to consist of 10 graduated leaves and 2 full length leaves. The spring is to support a central load of 5 kN over a span of 1100 mm with the central band width of 100 mm. The ratio of total depth of spring to its width is to be 2.5. Determine the width and thickness of leaves limiting the maximum equalized stress induced in the leaves to 350 MPa. Also determine the initial gap to be provided between the full length and graduated leaves before the assembly. **VTU, Jan./Feb. 2005**

9. a) Design a helical compression spring required for a spring loaded safety value mounted on a pressure vessel. The spring is subjected to an initial compression of 50 mm at the time of assembly and will open by 10 mm when the pressure approaches 6 MPa. The diameter of the valve is 25 mm.

   b) A semi elliptical laminated leaf spring with two full length leaves and ten graduated leaves are to be designed to support a central load of 6kN over two points 1000 mm apart. The central band width is 100 mm. The ratio of total depth of the spring to its width is 2.5. The design normal stress of the material of the leaves is 400 MPa and the Modulus of elasticity is 208 GPa. Determine:

   i) Width and thickness of leaves

   ii) The initial gap between full length and graduated leaves

   iii) The central bolt load. **VTU, Jan./Feb. 2006**

10. a. Explain about stress concentration in helical coil springs, how is it being taken care of?

    b. What is surging in springs and how it can be overcome?

    c. The spring used in an automobile engine has to exert 500N when the value is closed and 600N when the valve is open. The displacement of the valve is 5mm. The engine crankshaft rotates at 8000 rpm. Design the spring if permissible stress in the material of the spring is 300MPa. The ratio of mean coil diameter to the wire diameter is 6. The specific weight and the Modulus of rigidity of the spring material are 7.35×10^{-5} N/mm^3 and 8×10^4 MPa, respectively. The ends of the spring are square and ground. Inspect the suitability of the spring for this engine. At what speed of the engine does the spring resonate?

VTU, July 2006

11. a) What is surging in helical springs and how it can be eliminated?

    b) A railway wagon weighing 50 kN and moving with a speed of 8km/hr has to be stopped by four buffer springs in which the maximum compression allowed is 220mm. Find the number of active turns in each spring of mean diameter 150mm. The diameter of spring wire is 25mm. Also determine the maximum shear stress in each spring. Take G = 84 GPa.

    c) A locomotive spring has an overall length of 1100mm and sustains a load of 75 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100mm. All the leaves are to be stressed at 0.4GPa when fully loaded. The ratio of total depth of spring to its width is to be 2. Determine i) Width and thickness of leaves ii) The initial gap between full length and graduated leaves.

    Take E = 206.8 GPa. **VTU, Dec. 06/Jan. 07**

12. a) Derive the equation for energy stored in a helical spring.

    b) Design a helical spring for a spring loaded safety valve (Rams bottom safety valve) for the following conditions:

    Diameter of valve seat = 60 mm, Operating pressure = 0.7 N/mm^2, Maximum pressure when the valve blows off freely = 0.75 N/mm^2, Maximum lift of the valve when the pressure rises from 0.7 to 0.75 N/mm^2 = 3.5mm, Maximum allowablestress = 550 N/mm^2, Modulus of rigidity = 84 kN/mm^2, Spring index = 6. **VTU, July 2007**

13. a) Derive an expression for the stress induced in helical coil spring.

    b) Design a helical compression spring for a maximum load of 1000 N and for a deflection of 25mm. The maximum permissible shear stress for a spring wire is 420 N/mm^2, Modulus of rigidity is 0.84×10^5 N/mm^2 and value of spring index is 6.

    c) A one meter long Cantilever spring is composed of 8 graduated leaves and one extra full length leaf. The leaves are 45mm wide. A load of 2000 N at the end of the spring causes a deflection of 75 mm. Determine the thickness of the leaves and maximum bending stress in the full length leaf assuming that leaves are not prestressed.

    VTU, Dec. 07/Jan. 08

14. a) A railway wagon weighing 40 kN and moving with a speed of 10 km/hour has to be stopped by four buffer springs in which the max-compression allowed is 200mm, find the number of turns in each spring of mean diameter 150mm. The diameter of spring wire is 25mm. Take $G = 82.7 \times 10^3$ MPa.

    b) Design a truck spring that has 12 number of leaves two of which are full length leaves. The spring supports are 1 meter apart and the central band is 70 mm wide. The central load is to be 6 kN with a permissible stress of 200 MPa. Determine the thickness, width and deflection of spring leaves if the ratio of total depth to width of spring is 3.

    VTU, June/July 2008

15. a) Derive an expression for the maximum strain energy stored in the closed coil spring under axial load in term of maximum shear stress, modulus of rigidity and volume of the spring.

b) An automobile helical coil spring is to have a mean diameter of 80m and stiffness 200N/mm. The total axial force is 8000 N and allowable shear stress of spring material is $320N/mm^2$ and $G = 8 \times 10^4 N/mm^2$. Calculate.

i) Diameter of coil, ii) Number of effective coils, iii) Free length of spring and iv) Maximum energy which can be stored in the spring.

c) A laminated semielliptical leaf spring under central load of 10kN is to have an effective length of 1m and not to deflect more than 75mm. The spring has 10 leaves, 2 of which are full length and have been pre-stressed so that all leaves have same stress at full load condition. All the leaves have same width and thickness. The maximum allowable stress in the leaves is $350N/mm^2$. Calculate the width and thickness of leaves.

VTU, Dec. 08/Jan. 09

UNIT 4

DESIGN OF SPUR AND HELICAL GEARS

4.1 INTRODUCTION

Gears are used to transmit motion or power from one shaft to another preferably if the centre distance between the two shafts is small. It is a positive and smooth drive. It is possible to drive shafts that are parallel, intersecting, or neither parallel nor intersecting by the use of gears. The most commonly used types are (i) spur gear, (ii) Helical gear (iii) Bevel gear (iv) worm and worm wheel (v) Spiral gear (vi) Rack and pinion.

4.2 CLASSIFICATION

Gears are classified as follows.

i) According to the relative position of axes, gears are classified as parallel axes, intersecting axes and non intersecting and nonparallel axes gears.

   If the axes of the shaft on which the gears are mounted are parallel, it is called parallel axes gears.

   Example : Spur gear, Helical gear, Double Helical gear.

   If the axes of the shaft on which the gears are mounted are intersecting, it is called intersecting axes gears.

   Example : Bevel gear [straight, spiral or zerol]

   If the axes of the shaft on which the gears are mounted are neither parallel nor intersecting, it is called non parallel and non intersecting axes gears.

   Example : Crossed Helical gear, worm gear, Hypoid gears

ii) According to the peripheral velocity of gears, it is classified as low velocity, medium velocity and high velocity gears.

   If the speed is less than 3 m/sec, it is termed as low velocity gears. Similarly if the speed is between 3 to 15 m/sec it is termed as medium velocity gears and if it is more than 15 m/sec, it is called high velocity gear.

iii) According to the type of gearing, it is classified as internal gearing and external gearing.